Quadratic Equations

NCERT TEXTBOOK QUESTIONS SOLVED

EXERCISE 4.1

Q. 1. *Check whether the following are quadratic equations:*

(i) $(x + 1)^2 = 2(x - 3)$

(ii) $x^2 - 2x = (-2)(3 - x)$

(iii) $(x - 2)(x + 1) = (x - 1)(x + 3)$

(iv) $(x - 3)(2x + 1) = x(x + 5)$

(v) $(2x - 1)(x - 3) = (x + 5)(x - 1)$ (vi) $x^2 + 3x + 1 = (x - 2)^2$

(vii) $(x + 2)^3 = 2x(x^2 - 1)$ (viii) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

Sol. **(i) $(x + 1)^2 = 2(x - 3)$**

We have:

$$(x + 1)^2 = 2(x - 3)$$
$$\Rightarrow \quad x^2 + 2x + 1 = 2x - 6$$
$$\Rightarrow \quad x^2 + 2x + 1 - 2x + 6 = 0$$
$$\Rightarrow \quad x^2 + 7 = 0$$

Since $x^2 + 7$ is a quadratic polynomial

$\therefore$ $(x + 1)^2 = 2(x - 3)$ is a quadratic equation.

(ii) $x^2 - 2x = (-2)(3 - x)$

We have:

$$x^2 - 2x = (-2)(3 - x)$$
$$\Rightarrow \quad x^2 - 2x = -6 + 2x$$
$$\Rightarrow \quad x^2 - 2x - 2x + 6 = 0$$
$$\Rightarrow \quad x^2 - 4x + 6 = 0$$

Since $x^2 - 4x + 6$ is a quadratic polynomial

$\therefore$ $x^2 - 2x = (-2)(3 - x)$ is a quadratic equation.

(iii) $(x - 2)(x + 1) = (x - 1)(x + 3)$

We have:

$$(x - 2)(x + 1) = (x - 1)(x + 3)$$
$$\Rightarrow \quad x^2 - x - 2 = x^2 + 2x - 3$$
$$\Rightarrow \quad x^2 - x - 2 - x^2 - 2x + 3 = 0$$
$$\Rightarrow \quad -3x + 1 = 0$$

Since $-3x + 1$ is a linear polynomial

$\therefore$ $(x - 2)(x + 1) = (x - 1)(x + 3)$ is not quadratic equation.

(iv) $(x - 3)(2x + 1) = x(x + 5)$

We have:

$$(x - 3)(2x + 1) = x(x + 5)$$
$$\Rightarrow \quad 2x^2 + x - 6x - 3 = x^2 + 5x$$
$$\Rightarrow \quad 2x^2 - 5x - 3 - x^2 - 5x = 0$$
$$\Rightarrow \quad x^2 + 10x - 3 = 0$$

Since $x^2 + 10x - 3$ is a quadratic polynomial

$\therefore$ $(x - 3)(2x + 1) = x(x + 5)$ is a quadratic equation.

(v) $(2x - 1)(x - 3) = (x + 5)(x - 1)$

We have:

$$(2x - 1)(x - 3) = (x + 5)(x - 1)$$

$\Rightarrow \quad 2x^2 - 6x - x + 3 = x^2 - x + 5x - 5$

$\Rightarrow 2x^2 - x^2 - 6x - x + x - 5x + 3 + 5 = 0$

$\Rightarrow \quad x^2 - 11x + 8 = 0$

Since $x^2 - 11x + 8$ is a quadratic polynomial

$\therefore$ $(2x - 1)(x - 3) = (x + 5)(x - 1)$ is a quadratic equation.

(vi) $\mathbf{x^2 + 3x + 1 = (x - 2)^2}$

We have:

$x^2 + 3x + 1 = (x - 2)^2$

$\Rightarrow \quad x^2 + 3x + 1 = x^2 - 4x + 4$

$\Rightarrow \quad x^2 + 3x + 1 - x^2 + 4x - 4 = 0$

$\Rightarrow \quad 7x - 3 = 0$

Since $7x - 3$ is a linear polynomial.

$\therefore$ $x^2 + 3x + 1 = (x - 2)^2$ is not a quadratic equation.

(vii) $\mathbf{(x + 2)^3 = 2x(x^2 - 1)}$

We have:

$(x + 2)^3 = 2x(x^2 - 1)$

$\Rightarrow \quad x^3 + 3x^2(2) + 3x(2)^2 + (2)^3 = 2x^3 - 2x$

$\Rightarrow \quad x^3 + 6x^2 + 12x + 8 = 2x^3 - 2x$

$\Rightarrow x^3 + 6x^2 + 12x + 8 - 2x^3 + 2x = 0$

$\Rightarrow \quad -x^3 + 6x^2 + 14x + 8 = 0$

Since $-x^3 + 6x^2 + 14x + 8$ is a polynomial of degree 3

$\therefore$ $(x + 2)^3 = 2x(x^2 - 1)$ is not a quadratic equation.

(viii) $\mathbf{x^3 - 4x^2 - x + 1 = (x - 2)^3}$

We have:

$x^3 - 4x^2 - x + 1 = (x - 2)^3$

$\Rightarrow \quad x^3 - 4x^2 - x + 1 = x^3 + 3x^2(-2) + 3x(-2)^2 + (-2)^3$

$\Rightarrow \quad x^3 - 4x^2 - x + 1 = x^3 - 6x^2 + 12x - 8$

$\Rightarrow x^3 - 4x^2 - x - 1 - x^3 + 6x^2 - 12x + 8 = 0$

$\Rightarrow \quad 2x^2 - 13x + 9 = 0$

Since $2x^2 - 13x + 9$ is a quadratic polynomial

$\therefore$ $x^3 - 4x^2 - x + 1 = (x - 2)^3$ is a quadratic equation.

Q. 2. *Represent the following situations in the form of quadratic equations:*

(i) *The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.*

(ii) *The product of two consecutive positive integers is 306. We need to find the integers.*

(iii) *Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.*

(iv) *A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.*

Sol. (*i*) Let the breadth = x metres

$\because$ Length = 2 (Breadth) + 1

$\therefore$ Length $= (2x + 1)$ metres

Since Length × Breadth = Area

$$\therefore \quad (2x + 1) \times x = 528$$

$$\Rightarrow \quad 2x^2 + x = 528$$

$$\Rightarrow \quad 2x^2 + x - 528 = 0$$

Thus, the required quadratic equation is

$$\mathbf{2x^2 + x - 528 = 0}$$

(*ii*) Let the two consecutive numbers be x and $(x + 1)$.

$\because$ Product of the numbers = 306

$$\therefore \quad x(x + 1) = 306$$

$$\Rightarrow \quad x^2 + x = 306$$

$$\Rightarrow \quad x^2 + x - 306 = 0$$

Thus, the required equdratic equation is

$$\mathbf{x^2 + x - 306 = 0}$$

(*iii*) Let the present age = x

$\therefore$ Mother's age $= (x + 26)$ years

After 3 years

His age $= (x + 3)$ years

Mother's age $= [(x + 26) + 3]$ years $= (x + 29)$ years

According to the condition,

$$\begin{bmatrix}\text{Product of their ages} \\ \text{after 3 years}\end{bmatrix} = 360$$

$$\Rightarrow \quad (x + 3) \times (x + 29) = 360$$

$$\Rightarrow \quad x^2 + 29x + 3x + 87 = 360$$

$$\Rightarrow \quad x^2 + 29x + 3x + 87 - 360 = 0$$

$$\Rightarrow \quad x^2 + 32x - 273 = 0$$

Thus, the required quadratic equation is

$$x^2 + 32x - 273 = 0$$

(*iv*) Let the speed of the train = u km/hr

Distance covered = 480 km

Time taken = Distance ÷ Speed

$= (480 \div u)$ hours

$= \frac{480}{u}$ hours

In second case,

Speed $= (u - 8)$ km/hour

$$\therefore \quad \text{Time taken} = \frac{\text{Distance}}{\text{speed}} = \frac{480}{(u-8)} \text{ hours}$$

According to the condition,

$$\frac{480}{u-8} - \frac{480}{u} = 3$$

$\Rightarrow \quad 480u - 480(u - 8) = 3u(u - 8)$

$\Rightarrow \quad 480u - 480u + 3840 = 3u^2 - 24u$

$\Rightarrow \quad 3840 - 3u^2 + 24u = 0$

$\Rightarrow \quad 1280 - u^2 + 8u = 0$

$\Rightarrow \quad -1280 + u^2 - 8u = 0$

$\Rightarrow \quad u^2 - 8u - 1280 = 0$

Thus, the required quadratic equation is

$$u^2 - 8u - 1280 = 0$$

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EXERCISE 4.2

Q. 1. *Find the roots of the following quadratic equations by factorisation:*

(i) $x^2 - 3x - 10 = 0$ *(ii)* $2x^2 + x - 6 = 0$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ *(iv)* $2x^2 - x + \frac{1}{8} = 0$

(v) $100x^2 - 20x + 1 = 0$

Sol. ***(i)*** $\mathbf{x^2 - 3x - 10 = 0}$

We have:

$x^2 - 3x - 10 = 0$

$\Rightarrow \quad x^2 - 5x + 2x - 10 = 0$

$\Rightarrow \quad x(x - 5) + 2(x - 5) = 0$

$\Rightarrow \quad (x - 5)(x + 2) = 0$

Either $x - 5 = 0 \Rightarrow x = 5$

or $x + 2 = 0 \Rightarrow x = -2$

Thus, the required roots are $\mathbf{x = 5}$ **and** $\mathbf{x = -2}$.

(ii) $\mathbf{2x^2 + x - 6 = 0}$

We have:

$2x^2 + x - 6 = 0$

$\Rightarrow \quad 2x^2 + 4x - 3x - 6 = 0$

$\Rightarrow \quad 2x(x + 2) - 3(x + 2) = 0$

$\Rightarrow \quad (x + 2)(2x - 3) = 0$

Either $x + 2 = 0 \Rightarrow x = -2$

or $2x - 3 = 0 \Rightarrow x = \frac{3}{2}$

Thus, the required roots are $\mathbf{x = -2}$ **and** $\mathbf{x = \frac{3}{2}}$.

(iii) $\mathbf{\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0}$

We have:

$$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$$

$$\Rightarrow \quad \sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$$

$$\Rightarrow \quad \sqrt{2}x^2 + \left(\sqrt{2}\cdot\sqrt{2}\right)x + 5x + 5\cdot\sqrt{2} = 0$$

$$\Rightarrow \quad \sqrt{2}x\left[x+\sqrt{2}\right] + 5\left[x+\sqrt{2}\right] = 0$$

$$\Rightarrow \quad \left(x+\sqrt{2}\right)\left(\sqrt{2}x+5\right) = 0$$

Either $x+\sqrt{2} = 0 \Rightarrow x = -\sqrt{2}$

or $\sqrt{2}x+5 = 0 \Rightarrow x = -\dfrac{5}{\sqrt{2}}$

Thus, the required roots are $\boldsymbol{x = -\sqrt{2}}$ and $\boldsymbol{x = \dfrac{-5}{\sqrt{2}}}$.

(*iv*) $\mathbf{2x^2 - x + \dfrac{1}{8} = 0}$

We have:

$$2x^2 - x + \frac{1}{8} = 0$$

$$\Rightarrow \quad 16x^2 - 8x + 1 = 0$$

$$\Rightarrow \quad 16x^2 - 4x - 4x + 1 = 0$$

$$\Rightarrow \quad 4x(4x-1) - 1(4x-1) = 0$$

$$\Rightarrow \quad (4x-1)(4x-1) = 0$$

$$\Rightarrow \quad x = \frac{1}{4} \text{ and } x = \frac{1}{4}$$

Thus, the required roots are $\boldsymbol{x = \dfrac{1}{4}}$ and $\boldsymbol{x = \dfrac{1}{4}}$.

(*v*) $\mathbf{100x^2 - 20x + 1 = 0}$

We have:

$$100x^2 - 20x + 1 = 0$$

$$\Rightarrow \quad 100x^2 - 10x - 10x + 1 = 0$$

$$\Rightarrow \quad 10x(10x-1) - 1(10x-1) = 0$$

$$\Rightarrow \quad (10x-1)(10x-1) = 0$$

$$\Rightarrow \quad (10x-1) = 0 \text{ and } (10x-1) = 0$$

$$\Rightarrow \quad x = \frac{1}{10} \text{ and } x = \frac{1}{10}$$

Thus, the required roots are $\boldsymbol{x = \dfrac{1}{10}}$ and $\boldsymbol{x = \dfrac{1}{10}}$.

Q. 2. *Solve the problems given in Example 1.*

Sol. (*i*) We have:

$$x^2 - 45x + 324 = 0$$

$\because$ *$(-9)\times(-36) = 324$ and $(-9)+(-36) = -45$*

$$\Rightarrow \quad x^2 - 9x - 36x + 324 = 0$$

$$\Rightarrow \quad x(x-9) - 36(x-9) = 0$$

$$\Rightarrow \quad (x-9)(x-36) = 0$$

Either $x - 9 = 0 \Rightarrow x = 9$

or $x - 36 = 0 \Rightarrow x = 36$

Thus, $\boldsymbol{x = 9}$ and $\boldsymbol{x = 36}$

(*ii*) We have:

$$x^2 - 55x + 750 = 0$$

$$\Rightarrow \quad x^2 - 30x - 25x + 750 = 0$$

$\because$ *(– 30) × (– 25) = 750 and (– 30) + (– 25) = – 55*

$$\Rightarrow \quad x(x - 30) - 25(x - 30) = 0$$

$$\Rightarrow \quad (x - 30)(x - 25) = 0$$

Either $x - 30 = 0 \Rightarrow x = 30$

or $x - 25 = 0 \Rightarrow x = 25$

Thus, $\mathbf{x = 30}$ and $\mathbf{x = 25.}$

Q. 3. *Find two numbers whose sum is 27 and product is 182.*

Sol. Here, sum of the numbers is 27.

Let one of the numbers be x.

$\therefore$ Other number = $27 - x$

According to the condition,

Product of the numbers = 182

$$\Rightarrow \quad x(27 - x) = 182$$

$$\Rightarrow \quad 27x - x^2 = 182$$

$$\Rightarrow \quad -x^2 + 27x - 182 = 0$$

$$\Rightarrow \quad x^2 - 27x + 182 = 0$$

$$\Rightarrow \quad x^2 - 13x - 14x + 182 = 0$$

$\because$ *– 27 = (– 13) + (– 14) and (– 13) × (– 14) = 182*

$$\Rightarrow \quad x(x - 13) - 14(x - 13) = 0$$

$$\Rightarrow \quad (x - 13)(x - 14) = 0$$

Either $x - 13 = 0 \Rightarrow x = 13$

or $x - 14 = 0 \Rightarrow x = 14$

Thus, the required numbers are **13** and **14.**

Q. 4. *Find two consecutive positive integers, sum of whose squares is 365.*

Sol. Let the two consecutive positive integers be x and $(x + 1)$.

Since the sum of the squares of the numbers = 365

$$\therefore \quad x^2 + (x + 1)^2 = 365$$

$$\Rightarrow \quad x^2 + [x^2 + 2x + 1] = 365$$

$$\Rightarrow \quad x^2 + x^2 + 2x + 1 = 365$$

$$\Rightarrow \quad 2x^2 + 2x + 1 - 365 = 0$$

$$\Rightarrow \quad 2x^2 + 2x - 364 = 0$$

$$\Rightarrow \quad x^2 + x - 182 = 0$$

$$\Rightarrow \quad x^2 + 14x - 13x - 182 = 0$$

$\because$ *+ 14 – 13 = 1 and 14 × (– 13) = – 182*

$$\Rightarrow \quad x(x + 14) - 13(x + 14) = 0$$

$$\Rightarrow \quad (x + 14)(x - 13) = 0$$

Either $x + 14 = 0 \Rightarrow x = -14$

or $x - 13 = 0 \Rightarrow x = 13$

Since x has to be a positive integer

$$\therefore \quad x = 13$$

$$\Rightarrow \quad x + 1 = 13 + 1 = 14$$

Thus, the required consecutive positive integers are **13** and **14.**

Q. 5. *The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.*

Sol. Let the base of the given right triangle be 'x' cm.

$\therefore$ Its height $= (x - 7)$ cm

$\because$ Hypotenuse $= \sqrt{(\text{Base})^2 + (\text{Height})^2}$

$\therefore \quad 13 = \sqrt{x^2 + (x-7)^2}$

Squaring both sides, we get

$$169 = x^2 + (x - 7)^2$$
$$\Rightarrow \quad 169 = x^2 + x^2 - 14x + 49$$
$$\Rightarrow \quad 2x^2 - 14x + 49 - 169 = 0$$
$$\Rightarrow \quad 2x^2 - 14x - 120 = 0$$
$$\Rightarrow \quad x^2 - 7x - 60 = 0$$
$$\Rightarrow \quad x^2 - 12x + 5x - 60 = 0$$
$$\Rightarrow \quad x(x - 12) + 5(x - 12) = 0$$
$$\Rightarrow \quad (x - 12)(x + 5) = 0$$

$\because$ $(-12) \times 5 = -60$ and $(-12) + 5 = -7$

Either $x - 12 = 0 \Rightarrow x = 12$

or $x + 5 = 0 \Rightarrow x = -5$

But the side of triangle can never be negative,

$\Rightarrow \quad x = 12$

$\therefore$ Length of the base $= 12$ cm

$\Rightarrow$ Length of the height $= (12 - 7)$ cm $= 5$ cm

Thus, the required base = **12 cm** and height = **5 cm.**

Q. 6. *A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was ₹ 90, find the number of articles produced and the cost of each article.*

Sol. Let the number of articles produced in a day $= x$

$\therefore$ Cost of production of each article = ₹ $(2x + 3)$

According to the condition,

$$\text{Total cost} = 90$$
$$\Rightarrow \quad x \times (2x + 3) = 90$$
$$\Rightarrow \quad 2x^2 + 3x = 90$$
$$\Rightarrow \quad 2x^2 + 3x - 90 = 0$$
$$\Rightarrow \quad 2x^2 - 12x + 15x - 90 = 0$$
$$\Rightarrow \quad 2x(x - 6) + 15(x - 6) = 0$$
$$\Rightarrow \quad (x - 6)(2x + 15) = 0$$

Either $x - 6 = 0 \Rightarrow x = 6$

or $2x + 15 = 0 \Rightarrow x = \dfrac{-15}{2}$

But the number of articles cannot be negative.

$\therefore \quad x = \dfrac{-15}{2}$ is not required

$\Rightarrow \qquad x = 6$

$\therefore$ Cost of each article = ₹ (2 × 6 + 3) = ₹ 15

Thus, the required number of articles produced is **6** and the cost of each article is **₹ 15.**

- **Solving a quadratic equation by completing the square:**

Example: Solve $x^2 + 4x - 5 = 0$.

$$x^2 + 4x - 5 = 0$$
$$\Rightarrow \quad x^2 + (2)(2)(1)x - 5 = 0$$
$$\Rightarrow \quad x^2 + (2)(2)(1)x + (2)^2 - (2)^2 - 5 = 0$$
$$\Rightarrow \quad (x+2)^2 - 9 = 0$$
$$\Rightarrow \quad (x+2)^2 = 9$$
$$\Rightarrow \quad x + 2 = \pm 3$$
$$\Rightarrow \quad x = 3 - 2 \quad \text{or} \quad x = -3 - 2$$
$$\Rightarrow \quad x = 1 \quad \text{or} \quad x = -5$$

- **Steps involved in solving $ax^2 + bx + c = 0$**

1. Divide the equation by a:

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

2. Write $\frac{b}{a}$ as $2\left(\frac{b}{2a}\right)(1)$, and c as $\left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + c$.

3. Therefore, the equation becomes:

$$x^2 + 2\left(\frac{b}{2a}\right)(1) + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + c = 0$$

Here, the first three terms are a perfect square.

$$\Rightarrow \left(x + \frac{b}{2a}\right)^2 - \left[\left(\frac{b}{2a}\right)^2 - c\right] = 0$$

4. Now this can be solved easily to first find the value of $x + \frac{b}{2a}$ and then x.

- **Quadratic Formula:**

The roots of $ax^2 + bx + c = 0$ are also given simply by the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

i.e.,
$$x = \frac{-b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a}$$

or
$$x = \frac{-b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}$$

The term $b^2 - 4ac$ is said to be the discriminant D.

(*i*) If $D > 0$, roots exist and they are distinct.

(*ii*) If $D = 0$, the two roots are equal and real.

(*iii*) If $D < 0$, $\sqrt{b^2 - 4ac}$ does not exist, so there are no real roots.

NCERT TEXTBOOK QUESTIONS SOLVED

EXERCISE 4.3

Q. 1. *Find the roots of the following quadratic equations, if they exist, by the method of completing the square:*

(i) $2x^2 - 7x + 3 = 0$ (ii) $2x^2 + x - 4 = 0$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$ (iv) $2x^2 + x + 4 = 0$

Sol. **(i) $2x^2 - 7x + 3 = 0$**

Dividing throughout by the co-efficient of x^2, we get

$$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

$$\Rightarrow \left\{x - \frac{1}{2}\left(\frac{7}{2}\right)\right\}^2 - \left(\frac{7}{4}\right)^2 + \frac{3}{2} = 0$$

$$\Rightarrow \left\{x - \frac{7}{4}\right\}^2 - \frac{49}{16} + \frac{3}{2} = 0$$

$$\Rightarrow \left\{x - \frac{7}{4}\right\}^2 - \frac{49}{16} + \frac{24}{16} = 0$$

$$\Rightarrow \left\{x - \frac{7}{4}\right\}^2 - \frac{25}{16} = 0$$

$$\Rightarrow \left\{x - \frac{7}{4}\right\}^2 = \frac{25}{16} = \left(\frac{5}{4}\right)^2$$

$$\Rightarrow x - \frac{7}{4} = \pm\frac{5}{4}$$

Case I:

When $\frac{5}{4}$ is $+ve$, then

$$x - \frac{7}{4} = \frac{5}{4} \Rightarrow x = \frac{5}{4} + \frac{7}{4}$$

$$\Rightarrow x = \frac{12}{4} = 3$$

Case II:

When $\frac{5}{4}$ is $-ve$, then

$$x - \frac{7}{4} = -\frac{5}{4} \Rightarrow x = \frac{-5}{4} + \frac{7}{4}$$

$$\Rightarrow x = \frac{2}{4} = \frac{1}{2}$$

Thus, required roots are

$$\mathbf{x = 3 \text{ and } x = \frac{1}{2}}$$

(*ii*) $\mathbf{2x^2 + x - 4 = 0}$

We have:

$$2x^2 + x - 4 = 0$$

Dividing throughout by 2,

$$x^2 + \frac{x}{2} - 2 = 0$$

$$\Rightarrow \left\{x + \frac{1}{2}\cdot\frac{1}{2}\right\}^2 - \left(\frac{1}{4}\right)^2 - 2 = 0$$

$$\Rightarrow \left(x + \frac{1}{4}\right)^2 - \frac{1}{16} - 2 = 0$$

$$\Rightarrow \left(x + \frac{1}{4}\right)^2 - \frac{33}{16} = 0$$

$$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{33}{16} = \left(\frac{\sqrt{33}}{4}\right)^2$$

$$\Rightarrow \left(x + \frac{1}{4}\right) = \pm\frac{\sqrt{33}}{4}$$

Case I:

When $\frac{\sqrt{33}}{4}$ is positive, then

$$x + \frac{1}{4} = \frac{\sqrt{33}}{4} \quad \Rightarrow \quad x = \frac{\sqrt{33}}{4} - \frac{1}{4}$$

$$\Rightarrow \quad x = \frac{\sqrt{33} - 1}{4}$$

Case II:

When $\frac{\sqrt{33}}{4}$ is negative, then

$$x + \frac{1}{4} = -\frac{\sqrt{33}}{4} \quad \Rightarrow \quad x = \frac{-\sqrt{33}}{4} - \frac{1}{4}$$

$$\Rightarrow x = \frac{-\sqrt{33} - 1}{4}$$

Thus, the required roots are

$$\mathbf{x = \frac{\sqrt{33} - 1}{4} \quad \text{and} \quad x = \frac{-\sqrt{33} - 1}{4}}$$

(*iii*) $\mathbf{4x^2 + 4\sqrt{3}\,x + 3 = 0}$

Dividing throughout by 4, we have:

$$x^2 + \sqrt{3}x + \frac{3}{4} = 0$$

$$\Rightarrow \left[x+\left(\frac{1}{2}\cdot\sqrt{3}\right)\right]^2 - \left(\frac{1}{2}\sqrt{3}\right)^2 + \frac{3}{4} = 0$$

$$\Rightarrow \left[x+\frac{\sqrt{3}}{2}\right]^2 - \frac{3}{4} + \frac{3}{4} = 0$$

$$\Rightarrow \left[x+\frac{\sqrt{3}}{2}\right]^2 = 0$$

$$\Rightarrow \left(x+\frac{\sqrt{3}}{2}\right)\left(x+\frac{\sqrt{3}}{2}\right) = 0$$

$$\Rightarrow x = -\frac{\sqrt{3}}{2} \text{ and } x = -\frac{\sqrt{3}}{2}$$

(*iv*) $2x^2 + x + 4 = 0$

Dividing throughout by 2, we have:

$$x^2 + \frac{x}{2} + 2 = 0$$

$$\Rightarrow \left[x+\frac{1}{4}\right]^2 - \left(\frac{1}{4}\right)^2 + 2 = 0$$

$$\Rightarrow \left[x+\frac{1}{4}\right]^2 - \frac{1}{16} + 2 = 0$$

$$\Rightarrow \left[x+\frac{1}{4}\right]^2 + \frac{31}{16} = 0$$

$$\Rightarrow \left[x+\frac{1}{4}\right]^2 = -\frac{31}{16}$$

But the square of a number cannot be negative.

$\therefore \left[x+\frac{1}{4}\right]^2$ cannot be a real value.

So, no real roots exist.

$\Rightarrow$ There is no real value of x satisfying the given equation.

Q. 2. *Find the roots of the following quadratic equations, using the quadratic formula:*

(*i*) $2x^2 - 7x + 3 = 0$

(*ii*) $2x^2 + x - 4 = 0$

(*iii*) $4x^2 + 4\sqrt{3}\,x + 3 = 0$

(*iv*) $2x^2 + x + 4 = 0$

Sol. **(*i*) $2x^2 - 7x + 3 = 0$**

Comparing the given equation with $ax^2 + bx + c = 0$, we have:

$a = 2$

$b = -7$

$c = 3$

$\therefore \quad b^2 - 4ac = (-7)^2 - 4(2)(3)$

$= 49 - 24 = 25 \geq 0$

Since $b^2 - 4ac > 0$

$\therefore$ The given equation has real roots. The roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-(-7) \pm \sqrt{25}}{2(2)}$$

$$= \frac{7 \pm 5}{4}$$

Taking $+ve$ sign,

$$x = \frac{7+5}{4} = \frac{12}{4} = 3$$

Taking $-ve$ sign,

$$x = \frac{7-5}{4} = \frac{2}{4} = \frac{1}{2}$$

Thus, the roots of the given equation are

$$\mathbf{x = 3 \quad \text{and} \quad x = \frac{1}{2}}$$

***(ii)* $2x^2 + x - 4 = 0$**

Comparing the given equation with $ax^2 + bx + c = 0$ we have:

$a = 2$

$b = 1$

$c = -4$

$\therefore \quad b^2 - 4ac = (1)^2 - 4(2)(-4)$

$= 1 + 32$

$= 33 > 0$

Since $b^2 - 4ac > 0$

$\therefore$ The given equation has equal roots. The roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-1 \pm \sqrt{33}}{2(2)} = \frac{-1 \pm \sqrt{33}}{4}$$

Taking $+ve$ sign,

$$x = \frac{-1 + \sqrt{33}}{4}$$

Taking $-ve$ sign,

$$x = \frac{-1 - \sqrt{33}}{4}$$

Thus, the required roots are:

$$x = \frac{-1+\sqrt{33}}{4} \quad \text{and} \quad x = \frac{-1-\sqrt{33}}{4}$$

(iii) $4x^2 + 4\sqrt{3}\,x + 3 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have:

$$a = 4$$
$$b = 4\sqrt{3}$$
$$c = 3$$

$$\therefore \quad b^2 - 4ac = \left(4\sqrt{3}\right)^2 - 4(4)(3)$$
$$= [16 \times 3] - 48$$
$$= 48 - 48 = 0$$

Since $b^2 - 4ac = 0$

$\therefore$ The given equation has real roots, which are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-4\sqrt{3} \pm \sqrt{0}}{2(4)}$$

$$= \frac{-4\sqrt{3} \pm 0}{8} = \frac{-\sqrt{3} \pm 0}{2}$$

$$\therefore \quad x = \frac{-\sqrt{3}}{2} \quad \text{and} \quad x = \frac{-\sqrt{3}}{2}$$

(iv) $2x^2 + x + 4 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have:

$$a = 2$$
$$b = 1$$
$$c = 4$$

$$\therefore \quad b^2 - 4ac = (1)^2 - 4(2)(4)$$
$$= 1 - 32 = -31 < 0$$

Since $b^2 - 4ac$ is less than 0, therefore the given equation does not have real roots.

Q. 3. *Find the roots of the following equations:*

(i) $x - \frac{1}{x} = 3,\ x \neq 0$ (ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30},\ x \neq -4, 7$ [CBSE 2012]

Sol. (i) $x - \frac{1}{x} = 3,\ x \neq 0$

Here, we have:

$$x - \frac{1}{x} = 3$$

$$\Rightarrow \quad x^2 - 1 = 3x$$

$\Rightarrow \quad x^2 - 3x - 1 = 0$ ...(1)

Comparing (1) with $ax^2 + bx + c = 0$, we have:

$$a = 1$$
$$b = -3$$
$$c = -1$$

$$\therefore \quad b^2 - 4ac = (-3)^2 - 4(1)(-1)$$
$$= 9 + 4 = 13 > 0$$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-(-3) \pm \sqrt{13}}{2(1)}$$

$$= \frac{3 \pm \sqrt{13}}{2}$$

Now, taking $+ve$ sign,

$$x = \frac{3 + \sqrt{13}}{2}$$

Taking $-ve$ sign,

$$x = \frac{3 - \sqrt{13}}{2}$$

Thus, the required roots of the given equation are:

$$\boldsymbol{x = \frac{3 + \sqrt{13}}{2} \quad \text{and} \quad x = \frac{3 - \sqrt{13}}{2}}$$

(*ii*) $\boldsymbol{\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30};\ x \neq -4,\ 7}$

We have:

$$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$$

$$\Rightarrow \quad (x - 7) - (x + 4) = \frac{11}{30}(x+4)(x-7)$$

$$\Rightarrow \quad x - 7 - x - 4 = \frac{11}{30}(x^2 - 3x - 28)$$

$$\Rightarrow \quad -11 \times 30 = 11(x^2 - 3x - 28)$$

$$\Rightarrow \quad -30 = x^2 - 3x - 28$$

$$\Rightarrow \quad x^2 - 3x - 28 + 30 = 0$$

$\Rightarrow \quad x^2 - 3x + 2 = 0$ ...(1)

Comparing (1) with $ax^2 + bx + c = 0$, we have:

$$a = 1$$
$$b = -3$$
$$c = 2$$

$\therefore \quad b^2 - 4ac = (-3)^2 - 4(1)(2)$
$= 9 - 8 = 1 > 0$

$\therefore$ The quadratic equation (1) has real roots, which are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-(-3) \pm \sqrt{1}}{2(1)}$$

$$= \frac{3 \pm 1}{2}$$

Taking +*ve* sign, we have:

$$x = \frac{3+1}{2} = \frac{4}{2} = 2$$

Taking –*ve* sign, we have:

$$x = \frac{3-1}{2} = 1$$

Thus, the roots of the given equation are:

$x = 2$ and $x = 1$.

Q. 4. *The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.*

Sol. Let the present age of Rehman = x

$\therefore$ 3 years ago Rehman's age = $(x - 3)$ years

5 years later Rehman's age = $(x + 5)$ years

Now, according to the condition,

$$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$

$$\Rightarrow \frac{(x+5)+(x-3)}{(x-3)(x+5)} = \frac{1}{3}$$

$$\Rightarrow 3[x + 5 + x - 3] = (x - 3)(x + 5)$$

$$\Rightarrow 3[2x + 2] = x^2 + 2x - 15$$

$$\Rightarrow 6x + 6 = x^2 + 2x - 15$$

$$\Rightarrow x^2 + 2x - 6x - 15 - 6 = 0$$

$$\Rightarrow x^2 - 4x - 21 = 0 \quad \text{...(1)}$$

Now, comparing (1) with $ax^2 + bx + c = 0$, we have:

$a = 1$
$b = -4$
$c = -21$

$\therefore \quad b^2 - 4ac = (-4)^2 - 4(1)(-21)$
$= 16 + 84$
$= 100$

Since, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow$ $x = \dfrac{-(-4) \pm \sqrt{100}}{2(1)}$

$= \dfrac{4 \pm 10}{2}$

Taking positive sign, we have:

$x = \dfrac{4 + 10}{2} = \dfrac{14}{2} = 7$

Taking negative sign, we have:

$x = \dfrac{4 - 10}{2} = \dfrac{-6}{2} = -3$

Since age cannot be negative,

$\therefore$ $x \neq -3 \Rightarrow x = 7$

So, the present age of Rehman = **7 years.**

Q. 5. *In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210? Find her marks in the two subjects.* [CBSE 2012]

Sol. Let, Shefali's marks in Mathematics = x

$\therefore$ Marks in English = $(30 - x)$ [$\because$ *Sum of their marks in Eng. and Maths = 30*]

Now, according to the condition,

$$(x + 2) \times [(30 - x) - 3] = 210$$

$$\Rightarrow (x + 2) \times (30 - x - 3) = 210$$

$$\Rightarrow (x + 2)(-x + 27) = 210$$

$$\Rightarrow -x^2 + 25x + 54 = 210$$

$$\Rightarrow -x^2 + 25x + 54 - 210 = 0$$

$$\Rightarrow -x^2 + 25x - 156 = 0$$

$$\Rightarrow x^2 - 25x + 156 = 0 \quad ...(1)$$

Now, comparing (1) with $ax^2 + bx + c = 0$

$a = 1$

$b = -25$

$c = 156$

$\therefore$ $b^2 - 4ac = (-25)^2 - 4(1)(156)$

$= 625 - 624 = 1$

Since, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \quad x = \dfrac{-(-25) \pm \sqrt{1}}{2(1)}$

$\Rightarrow \quad x = \dfrac{25 \pm 1}{2}$

Taking +*ve* sign, we have

$$x = \frac{25+1}{2} = \frac{26}{2} = 13$$

Taking – *ve* sign, we get

$$x = \frac{25-1}{2} = \frac{24}{2} = 12$$

When $x = 13$, then $30 - 13 = 17$

When $x = 12$, then $30 - 12 = 18$

Thus, marks in **Maths = 13,** marks in **English = 17**

marks in **Maths = 12,** marks in **English = 18**

Q. 6. *The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.*

Sol. Let the shorter side (*i.e.*, breadth) = x metres.

∴ The longer side (length) = $(x + 30)$ metres.

In a rectangle,

$$\text{diagonal} = \sqrt{(\text{breadth})^2 + (\text{length})^2}$$

$\Rightarrow \quad x + 60 = \sqrt{x^2 + (x+30)^2}$

$\Rightarrow \quad x + 60 = \sqrt{x^2 + x^2 + 60x + 900}$

$\Rightarrow \quad (x + 60)^2 = 2x^2 + 60x + 900$

$\Rightarrow \quad x^2 + 120x + 3600 = 2x^2 + 60x + 900$

$\Rightarrow \quad 2x^2 - x^2 + 60x - 120x + 900 - 3600 = 0$

$\Rightarrow \quad x^2 - 60x - 2700 = 0 \qquad ...(1)$

Comparing (1) with $ax^2 + bx + c = 0$

∴ $a = 1$

$b = -60$

$c = -2700$

∴ $b^2 - 4ac = (-60)^2 - 4(1)(-2700)$

$\Rightarrow \quad b^2 - 4ac = 3600 + 10800$

$\Rightarrow \quad b^2 - 4ac = 14400$

Since, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \quad x = \dfrac{-(-60) \pm \sqrt{14400}}{2(1)}$

$$\Rightarrow \quad x = \frac{60 \pm 120}{2}$$

Taking +*ve* sign, we get

$$x = \frac{60 + 120}{2} = \frac{180}{2} = 90$$

Taking –*ve* sign,

$$x = \frac{60 - 120}{2} = \frac{-60}{2} = -30$$

Since breadth cannot be negative,

$\therefore \quad x \neq -30 \quad \Rightarrow \quad x = 90$

$\therefore \; x + 30 = 90 + 30 = 120$

Thus, the shorter side = **90 m**

The longer side = **120 m.**

Q. 7. *The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.*

Sol. Let the larger number be x.

Since,

$$(\text{smaller number})^2 = 8\ (\text{larger number})$$

$$\Rightarrow \quad (\text{smaller number})^2 = 8x$$

$$\Rightarrow \quad \text{smaller number} = \sqrt{8x}$$

Now, according to the condition,

$$x^2 - \left(\sqrt{8x}\right)^2 = 180$$

$$\Rightarrow \quad x^2 - 8x = 180$$

$$\Rightarrow \quad x^2 - 8x - 180 = 0 \qquad ...(1)$$

Comparing (1) with $ax^2 + bx + c = 0$, we have:

$a = 1$

$b = -8$

$c = -180$

$$\therefore \; b^2 - 4ac = (-8)^2 - 4\,(1)\,(-180)$$

$$= 64 + 720 = 784$$

$$\text{Since,} \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore \quad x = \frac{-(-8) \pm \sqrt{784}}{2\,(1)}$$

$$\Rightarrow \quad x = \frac{8 \pm 28}{2} \qquad \Big|\; \because \sqrt{784} = 28$$

Taking +*ve* sign, we get

$$x = \frac{8 + 28}{2} = \frac{36}{2} = 18$$

Taking –*ve* sign, we get

$$x = \frac{8-28}{2} = \frac{-20}{2} = -10$$

But $x = -10$ is not admissible,

∴ The smaller number = 18

$$\therefore \sqrt{8 \times 18} = \sqrt{144} = \pm 12$$

Thus, the larger number = 12 or – 12

Thus, the two numbers are:

18 and 12 or **18 and –12**

Q. 8. *A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.* [NCERT Exemplar]

Sol. Let the uniform speed of the train be x km/hr.

Since, time taken by the train = $\frac{\text{Distance}}{\text{Speed}}$

$$\Rightarrow \quad \text{time} = \frac{360}{x} \text{ hr}$$

When speed is 5 km/hr more then time is 1 hour less.

$$\Rightarrow \quad \frac{360}{x+5} = \frac{360}{x} - 1$$

$$\Rightarrow \quad \frac{360}{x+5} - \frac{360}{x} = -1$$

$$\Rightarrow \quad 360\left[\frac{1}{x+5} - \frac{1}{x}\right] = -1$$

$$\Rightarrow \quad \frac{1}{x+5} - \frac{1}{x} = \frac{-1}{360}$$

$$\Rightarrow \quad \frac{x-(x+5)}{x(x+5)} = \frac{-1}{360}$$

$$\Rightarrow \quad x - x - 5 = \frac{-1(x+5)x}{360}$$

$$\Rightarrow \quad -5 \times 360 = -1(x^2 + 5x)$$

$$\Rightarrow \quad -5 \times 360 = -x^2 - 5x \qquad \because 360 \times 5 = 1800$$

$$\Rightarrow \quad x^2 + 5x - 1800 = 0 \qquad ...(1)$$

Comparing (1) with

$$ax^2 + bx + c = 0$$

$$\Rightarrow \quad a = 1$$

$$b = 5$$

$$c = -1800$$

$\therefore \quad b^2 - 4ac = (5)^2 - 4\ (1)\ (-1800)$

$= 25 + 7200$

$= 7225$

Since $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \quad x = \dfrac{-5 \pm \sqrt{7225}}{2\,(1)}$

$\Rightarrow \quad x = \dfrac{-5 \pm 85}{2\,(1)}$ | $\because \sqrt{7225} = 85$

Taking $+ve$ sign, we get:

$x = \dfrac{-5 + 85}{2} = \dfrac{80}{2} = 40$

Taking $-ve$ sign, we get

$x = \dfrac{-5 - 85}{2} = \dfrac{-90}{2} = -45$

Since, the speed of a vehicle cannot be negative,

$\therefore \quad x \neq -45$

Thus, $x = 40 \Rightarrow$ speed of the train = **40 km/hr.**

Q. 9. *Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.*

Sol. Let the smaller tap fills the tank in x hours.

$\therefore$ The larger tap fills the tank $(x - 10)$ hours.

$\Rightarrow$ Time to fill the tank by smaller tap = $\dfrac{1}{x}$ hours.

Time to fill the tank by larger tap = $\dfrac{1}{x - 10}$ hours.

Since, the tank filled by the two taps together in 1 hour = $\dfrac{1}{x} + \dfrac{1}{x - 10} = \dfrac{x - 10 + x}{x\,(x - 10)}$

$= \dfrac{2x - 10}{x^2 - 10x}$

Now, according to the condition,

$\dfrac{75}{8}\left(\dfrac{2x - 10}{x^2 - 10x}\right) = 1$ | $\because 9\dfrac{3}{8} = \dfrac{75}{8}$

$\Rightarrow \quad \dfrac{75\,(2x - 10)}{8\left(x^2 - 10x\right)} = 1$

$\Rightarrow \quad \dfrac{150x - 750}{8x^2 - 80x} = 1$

$\Rightarrow \quad 8x^2 - 80x = 150x - 750$

$\Rightarrow \quad 8x^2 - 80x - 150x = -750$

$\Rightarrow \quad 8x^2 - 230x + 750 = 0$ ...(1)

Comparing (1) with $ax^2 + bx + c = 0$, we get

$a = 8$

$b = -230$

$c = 750$

$\therefore \quad b^2 - 4ac = (-230)^2 - 4 (8) (750)$

$= 52900 - 24000$

$= 28900$

Since $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \quad x = \dfrac{-(-230) \pm \sqrt{28900}}{2(8)}$

$\Rightarrow \quad x = \dfrac{230 \pm 170}{16}$ $\quad \because \sqrt{28900} = 170$

Taking, the +*ve* sign, we get

$x = \dfrac{230 + 170}{16} = \dfrac{400}{16} = 25$

Taking the –*ve* sign, we get

$x = \dfrac{230 - 170}{16} = \dfrac{60}{16} = \dfrac{15}{4}$

For $\quad x = \dfrac{15}{4}, \quad (x - 10) = \dfrac{15}{4} - 10 = \dfrac{-25}{4}$

which is not possible, [$\because$ Time cannot be negative]

$\therefore \quad x = 25$

$\Rightarrow \quad x - 10 = 25 - 10 = 15$

Thus, time to fill the tank by the smaller tap alone = **25 hours** and larger tap alone = **15 hours.**

Q. 10. *An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11 km/h more than that of the passenger train, find the average speed of the two trains.*

Sol. Let the average speed of the passenger train be x km/hr

$\therefore$ Average speed of the express train = $(x + 11)$ km/hr

Total distance covered = 132 km

Also, Time $= \dfrac{\text{Distance}}{\text{Speed}}$

Time taken by the passenger train $= \dfrac{132}{x}$ hour

Time taken by the express train $= \dfrac{132}{x+11}$ hour

According to the condition, we get

$$\frac{132}{x+11} = \left(\frac{132}{x}\right) - 1$$

$$\Rightarrow \quad \frac{132}{x+11} - \frac{132}{x} = -1$$

$$\Rightarrow \quad 132\left[\frac{1}{x+11} - \frac{1}{x}\right] = -1$$

$$\Rightarrow \quad 132\left[\frac{x-x-11}{x(x+11)}\right] = -1$$

$$\Rightarrow \quad 132\left[\frac{-11}{x^2+11x}\right] = -1$$

$$\Rightarrow \quad -11\ (132) = -1\ (x^2 + 11x)$$

$$\Rightarrow \quad -1452 = -1\ (x^2 + 11x)$$

$$\Rightarrow \quad x^2 + 11x - 1452 = 0 \qquad ...(1)$$

Comparing (1) with $ax^2 + bx + c = 0$,

$$a = 1$$

$$b = 11$$

$$c = -1452$$

$$\therefore \quad b^2 - 4ac = (11)^2 - 4\ (1)\ (-1452)$$

$$\Rightarrow \quad b^2 - 4ac = 121 + 5808 = 5929$$

Since, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-11 \pm \sqrt{5929}}{2\,(1)} = \frac{-11 \pm 77}{2} \qquad \because \sqrt{5929} = 77$$

$\therefore$ Taking +*ve* sign, we get

$$x = \frac{-11 + 77}{2} = \frac{66}{2} = 33$$

Taking $-ve$ sign, we get

$$x = \frac{-11-77}{2} = \frac{-88}{2} = -44$$

But average speed cannot be negative

$\therefore \quad x \neq -44$

$\therefore \quad x = 33$

$\Rightarrow$ Average speed of the passenger train = **33 km/hr**

$\therefore$ Average speed of the express train = $(x + 11)$ km/hr

= (33 + 11) km/hr

= **44 km/hr**

Q. 11. *Sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares.*

Sol. Let the side of the smaller square be x m.

$\Rightarrow$ Perimeter of the smaller square = $4x$ m.

$\therefore$ Perimeter of the larger square = $(4x + 24)$ m

$\Rightarrow$ Side of the larger square = $\frac{\text{Perimeter}}{4}$

$$= \frac{4x+24}{4} \text{ m} = \frac{4(x+6)}{4} \text{ m} = (x+6) \text{ m}$$

$\therefore$ Area of the smaller square = $(\text{side})^2 = (x)^2 \text{ m}^2$

Area of the larger square = $(x + 6)^2 \text{ m}^2$

According to the condition,

$$x^2 + (x+6)^2 = 468$$

$$\Rightarrow x^2 + x^2 + 12x + 36 = 468$$

$$\Rightarrow 2x^2 + 12x - 432 = 0$$

$$\Rightarrow x^2 + 6x - 216 = 0 \quad \text{...(1)}$$

Comparing (1) with $ax^2 + bx + c = 0$

$\therefore \quad a = 1$

$b = 6$

$c = -216$

$\therefore \quad b^2 - 4ac = (6)^2 - 4(1)(-216)$

$= 36 + 864 = 900$

Since, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore \quad x = \frac{-6 \pm \sqrt{900}}{2(1)}$

$\Rightarrow \quad x = \frac{-6 \pm 30}{2}$

Taking +*ve* sign, we have:

$$x = \frac{-6+30}{2} = \frac{24}{2} = 12$$

Taking –*ve* sign, we get

$$x = \frac{-6-30}{2} = \frac{-36}{2} = -18$$

But the length of a square cannot be negative

$\therefore \quad x = 12$

$\Rightarrow$ Length of the smaller square = **12 m**

Thus, the length of the larger square = $x + 6$

$= 12 + 6$

$=$ **18 m**

NCERT TEXTBOOK QUESTIONS SOLVED

EXERCISE 4.4

Q. 1. *Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:*

(i) $2x^2 - 3x + 5 = 0$ *(ii)* $3x^2 - 4\sqrt{3}\,x + 4 = 0$

(iii) $2x^2 - 6x + 3 = 0$

Sol. ***(i)*** $\mathbf{2x^2 - 3x + 5 = 0}$

Comparing the given quadratic equation with $ax^2 + bx + c = 0$, we have:

$a = 2$

$b = -3$

$c = 5$

$\therefore$ The discriminant $= b^2 - 4ac$

$= (-3)^2 - 4(2)(5)$

$= 9 - 40$

$= -31 < 0$

Since $b^2 - 4ac$ is negative.

$\therefore$ The given quadratic equation has **no real roots.**

(ii) $\mathbf{3x^2 - 4\sqrt{3}\,x + 4 = 0}$

Comparing the given quadratic equation with $ax^2 + bx + c = 0$, we get

$a = 3$

$b = -4\sqrt{3}$

$c = 4$

$\therefore \quad b^2 - 4ac = \left[-4\sqrt{3}\right]^2 - 4(3)(4)$

$= (16 \times 3) - 48$

$= 48 - 48 = 0$

Thus, the given quadratic equation has two real roots which are equal. Here, the roots are:

$$\frac{-b}{2a} \quad \text{and} \quad \frac{-b}{2a}$$

i.e., $$\frac{-(-4\sqrt{3})}{2 \times 3} \quad \text{and} \quad \frac{-(-4\sqrt{3})}{2 \times 3}$$

$$\Rightarrow \quad \frac{4\sqrt{3}}{2\sqrt{3} \times \sqrt{3}} \quad \text{and} \quad \frac{4\sqrt{3}}{2\sqrt{3} \times \sqrt{3}} \qquad \because 3 = \sqrt{3} \times \sqrt{3}$$

$$\Rightarrow \quad \frac{2}{\sqrt{3}} \quad \text{and} \quad \frac{2}{\sqrt{3}}$$

Thus, $x = \mathbf{\frac{2}{\sqrt{3}}}$ and $x = \mathbf{\frac{2}{\sqrt{3}}}$

(*iii*) $2x^2 - 6x + 3 = 0$

Comparing it with the general quadratic equation, we have:

$$a = 2$$
$$b = -6$$
$$c = 3$$

$$\therefore \quad b^2 - 4ac = (-6)^2 - 4(2)(3) = 36 - 24 = 12 > 0$$

$\therefore$ The given quadratic equation has two real and distinct roots, which are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-(-6) \pm \sqrt{12}}{2 \times 2}$$

$$= \frac{6 \pm 2\sqrt{3}}{4}$$

$$= \frac{3 \pm \sqrt{3}}{2}$$

Thus, the roots are:

$$x = \mathbf{\frac{3+\sqrt{3}}{2}} \quad \text{and} \quad x = \mathbf{\frac{3-\sqrt{3}}{2}}$$

Q. 2. *Find the values of k for each of the following quadratic equations, so that they have two equal roots:*

(*i*) $2x^2 + kx + 3 = 0$ (*ii*) $kx(x - 2) + 6 = 0$

Sol. **(*i*) $2x^2 + kx + 3 = 0$**

Comparing the given quadratic equation with $ax^2 + bx + c = 0$, we get

$$a = 2$$
$$b = k$$
$$c = 3$$

$\therefore \quad b^2 - 4ac = (-k)^2 - 4(2)(3)$
$= k^2 - 24$

$\because$ For a quadratic equation to have equal roots,

$b^2 - 4ac = 0$

$\therefore \quad k^2 - 24 = 0 \Rightarrow k = \pm\sqrt{24}$

$\Rightarrow \quad k = \pm 2\sqrt{6}$

Thus, the required values of k are

$\mathbf{2\sqrt{6}}$ and $\mathbf{-2\sqrt{6}}$

(*ii*) $\mathbf{kx(x-2)+6=0}$

Comparing $kx(x-2)+6=0$ *i.e.*, $kx^2 - 2kx + 6 = 0$ with $ax^2 + bx + c = 0$, we get

$a = k$
$b = -2k$
$c = 6$

$\therefore \quad b^2 - 4ac = (-2k)^2 - 4(k)(6)$
$= 4k^2 - 24k$

Since, the roots are real and equal,

$\therefore \quad b^2 - 4ac = 0$
$\Rightarrow \quad 4k^2 - 24k = 0$
$\Rightarrow \quad 4k(k-6) = 0$
$\Rightarrow \quad 4k = 0$ or $k - 6 = 0$
$\Rightarrow \quad k = 0$ or $k = 6$

But k cannot be 0, otherwise, the given equation is no more quadratic. Thus, the required value of $\mathbf{k = 6}$.

Q. 3. *Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m? If so, find its length and breadth.*

Sol. Let the breadth be x metres.

$\therefore$ Length $= 2x$ metres

Now, Area $=$ Length $\times$ Breadth
$= 2x \times x$ metre^2
$= 2x^2$ sq. metre.

According to the given condition,

$2x^2 = 800$

$\Rightarrow \quad x^2 = \frac{800}{2} = 400$

$\Rightarrow \quad x = \pm\sqrt{400} = \pm 20$

Therefore, $x = 20$ and $x = -20$

But $x = -20$ is possible | ($\because$ *breadth cannot be negative*).

$\therefore \quad x = 20$

$\Rightarrow \quad 2x = 2 \times 20 = 40$

Thus, **length = 40 m** and **breadth = 20 m**

Q. 4. *Is the following situation possible? If so, determine their present ages.*

The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Sol. Let the age of one friend = x years

$\therefore$ The age of the other friend = $(20 - x)$ years [$\because$ *Sum of their ages is 20 years*]

Four years ago

$$\text{Age of one friend} = (x - 4) \text{ years}$$

$$\text{Age of the other friend} = (20 - x - 4) \text{ years} = (16 - x) \text{ years}$$

According to the condition,

$$(x - 4) \times (16 - x) = 48$$

$$\Rightarrow \quad 16x - 64 - x^2 - 4x = 48$$

$$\Rightarrow \quad - x^2 - 20x - 64 - 48 = 0$$

$$\Rightarrow \quad - x^2 - 20x - 112 = 0$$

$$\Rightarrow \quad x^2 + 20x + 112 = 0 \quad \text{...(1)}$$

Here, $a = 1$, $b = 20$ and $c = 112$

$$\therefore \quad b^2 - 4ac = (20)^2 - 4\ (1)\ (112) = 400 - 448 = - 48 < 0$$

Since $b^2 - 4ac$ is less than 0.

$\therefore$ The quadratic equation (1) has no real roots.

Thus, the given equation is **not possible.**

Q. 5. *Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2? If so, find its length and breadth.*

Sol. Let the breadth of the rectangle be x metres.

Since, the parimetre of the rectangle = 80 m.

$$\therefore \quad 2\ [\text{Length} + \text{Breadth}] = 80$$

$$2\ [\text{Length} + x] = 80$$

$$\Rightarrow \quad \text{Length} + x = \frac{80}{2} = 40$$

$$\Rightarrow \quad \text{Length} = (40 - x) \text{ metres}$$

$$\therefore \quad \text{Area of the rectangle} = \text{Length} \times \text{breadth} = (40 - x) \times x \text{ sq. m} = 40x - x^2$$

Now, according to the given condition,

$$\text{Area of the rectangle} = 400 \text{ m}^2$$

$$\therefore \quad 40x - x^2 = 400$$

$$\Rightarrow \quad - x^2 + 40x - 400 = 0$$

$$\Rightarrow \quad x^2 - 40x + 400 = 0 \quad \text{...(1)}$$

Comparing (1) with $ax^2 + bx + c = 0$, we get

$$a = 1$$
$$b = -40$$
$$c = 400$$

$\therefore \quad b^2 - 4ac = (-40)^2 - 4(1)(400)$

$= 1600 - 1600 = 0$

Thus, the equation (1) has two equal and real roots.

$\because \quad x = \dfrac{-b}{2a}$ and $x = \dfrac{-b}{2a}$

$\therefore \quad \text{breadth} = \dfrac{-(-40)}{2(1)} = \dfrac{40}{2} = 20$

$\therefore \quad$ Breadth, $x = 20$ m

$\therefore \quad$ Length $= (40 - x) = (40 - 20)$ m $=$ **20 m.**

MORE QUESTIONS SOLVED

I. VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. *If the roots of the quadratic equation $-ax^2 + bx + c = 0$ are equal then show that $b^2 = 4ac$.*

Sol. $\because$ For equal roots, we have

$b^2 - 4ac = 0$

$\therefore \quad b^2 = 4ac$

Q. 2. *Find the value of 'k' for which the quadratic equation $kx^2 - 5x + k = 0$ have real roots.*

Sol. Comparing $kx^2 - 5x + k = 0$ with $ax^2 + bx + c = 0$, we have:

$$a = k$$
$$b = -5$$
$$c = k$$

$\therefore \quad b^2 - 4ac = (-5)^2 - 4(k)(k)$

$= 25 - 4k^2$

For equal roots, $b^2 - 4ac = 0$

$\therefore \quad 25 - 4k^2 = 0$

$\Rightarrow \quad 4k^2 = 25$

$\Rightarrow \quad k^2 = \dfrac{25}{4}$

$\Rightarrow \quad k = \pm\sqrt{\dfrac{25}{4}} = \pm\dfrac{5}{2}$

Q. 3. *If -4 is a root of the quadratic equation $x^2 + px - 4 = 0$ and $x^2 + px + k = 0$ has equal roots, find the value of k.*

Sol. $\because$ (-4) is a root of $x^2 + px - 4 = 0$

$\therefore \quad (-4)^2 + p(-4) = 0$

$\Rightarrow \quad 16 - 4p - 4 = 0$

$\Rightarrow \quad 4p = 12$ or $p = 3$

Now, $x^2 + px + k = 0$

$\Rightarrow \quad x^2 + 3x + k = 0$ $\quad [\because p = 3]$

Now, $a = 1$, $b = 3$ and $c = +k$

$\therefore \quad b^2 - 4ac = (3)^2 - 4(1)(k)$

$= 9 - 4k$

For equal roots, $b^2 - 4ac = 0$

$\Rightarrow \quad 9 - 4k = 0 \Rightarrow 4k = 9$

$\Rightarrow \quad \boldsymbol{k = \frac{9}{4}}$

Q. 4. *If one root of the quadratic equation $2x^2 - 3x + p = 0$ is 3, find the other root of the quadratic equation. Also find the value of p.*

Sol. We have:

$2x^2 - 3x + p = 0$ ...(1)

$\therefore \quad a = 2, \; b = -3$ and $c = p$

Since, the sum of the roots $= \frac{-b}{a}$

$= \frac{-(-3)}{2} = \frac{3}{2}$

$\because$ One of the roots $= 3$

$\therefore$ The other root $= \frac{3}{2} - 3 = \boldsymbol{\frac{-3}{2}}$

Now, substituting $x = 3$ in (1), we get

$2(3)^2 - 3(3) + p = 0$

$\Rightarrow \quad 18 - 9 + p = 0$

$\Rightarrow \quad 9 + p = 0 \Rightarrow \boldsymbol{p = -9}$

Q. 5. *If one of the roots of $x^2 + px - 4 = 0$ is -4 then find the product of its roots and the value of p.*

Sol. If -4 is a root of the quadratic equation,

$x^2 + px - 4 = 0$

$\therefore \; (-4)^2 + (-4)(p) - 4 = 0$

$\Rightarrow \quad 16 - 4p - 4 = 0$

$\Rightarrow \quad 12 - 4p = 0 \Rightarrow \boldsymbol{p = 3}$

Now, in $ax^2 + bx + c = 0$, the product of the roots $= \frac{c}{a}$

$\therefore$ Product of the roots in $x^2 - px - 4 = 0$

$= \frac{-4}{1} = \boldsymbol{-4}$

Q. 6. *For what value of k, does the given equation have real and equal roots?*
$(k + 1)\, x^2 - 2\,(k - 1)\, x + 1 = 0$.

Sol. Comparing the given equation with $ax^2 + bx + c = 0$, we have:

$$a = k + 1$$
$$b = -2\,(k - 1)$$
$$c = 1$$

For equal roots, $b^2 - 4ac = 0$

$\therefore \quad [-2\,(k-1)]^2 - 4\,(k+1)\,(1) = 0$

$\Rightarrow \quad 4\,(k-1)^2 - 4\,(k+1) = 0$

$\Rightarrow \quad 4\,(k^2 + 1 - 2k) - 4k - 4 = 0$

$\Rightarrow \quad 4k^2 + 4 - 8k - 4k - 4 = 0$

$\Rightarrow \quad 4k^2 - 12k = 0$

$\Rightarrow \quad 4k\,(k - 3) = 0$

$\Rightarrow \quad$ **$k = 0$ or $k = 3$**

Q. 7. *Using quadratic formula, solve the following quadratic equation for x:*

$$x^2 - 2ax + (a^2 - b^2) = 0$$

Sol. Comparing $x^2 - 2ax + (a^2 - b^2) = 0$, with $ax^2 + bx + c = 0$, we have:

$$a = 1, \quad b = -2a, \quad c = a^2 - b^2$$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-(-2a) \pm \sqrt{(-2a)^2 - 4\,(1)\left(a^2 - b^2\right)}}{2\,(1)}$$

$$= \frac{2a \pm \sqrt{4a^2 - 4a^2 + 4b^2}}{2}$$

$$= \frac{2a \pm \sqrt{4b^2}}{2} = \frac{2a \pm 2b}{2} = a \pm b$$

$\therefore \quad$ **$x = (a + b)$ or $x = (a - b)$**

Q. 8. *If one of the roots of the quadratic equation $2x^2 + kx - 6 = 0$ is 2, find the value of k. Also find the other root.*

Sol. Given equation:

$$2x^2 + kx - 6 = 0$$
$$\text{one root} = 2$$

Substituting $x = 2$ in $2x^2 + kx - 6 = 0$

We have:

$2\,(2)^2 + k\,(2) - 6 = 0$

$\Rightarrow \quad 8 + 2k - 6 = 0$

$\Rightarrow \quad 2k + 2 = 0 \Rightarrow$ **$k = -1$**

$\therefore \quad 2x^2 + kx - 6 = 0 \Rightarrow 2x^2 - x - 6 = 0$

$$\text{Sum of the roots} = \frac{-b}{a} = \frac{1}{2}$$

$$\therefore \quad \text{other root} = \frac{1}{2} - 2$$

$$= \mathbf{\frac{3}{2}}$$

Q. 9. *Determine the value of k for which the quadratic equation $4x^2 - 4kx + 1 = 0$ has equal roots.*

Sol. We have:

$$4x^2 - 4kx + 1 = 0$$

Comparing with $ax^2 + bx + c = 0$, we have

$$a = 4, \quad b = -4k \quad \text{and} \quad c = 1$$

$$\therefore \quad b^2 - 4ac = (-4k)^2 - 4(4k)(1)$$

$$= 16k^2 - 16$$

For equal roots

$$b^2 - 4ac = 0$$

$$\therefore \quad 16k^2 - 16 = 0$$

$$\Rightarrow \quad 16k^2 = 16 \quad \Rightarrow \quad k^2 = 1$$

$$\Rightarrow \quad k = \pm 1$$

Q. 10. *For what value of k, does the quadratic equation $x^2 - kx + 4 = 0$ have equal roots?*

Sol. Comparing $x^2 - kx + 4 = 0$ with $ax^2 + bx + c = 0$, we get

$$a = 1$$

$$b = -k$$

$$c = 4$$

$$\therefore \quad b^2 - 4ac = (-k)^2 - 4(1)(4) = k^2 - 16$$

For equal roots,

$$b^2 - 4ac = 0$$

$$\Rightarrow \quad k^2 - 16 = 0$$

$$\Rightarrow \quad k^2 = 16$$

$$\Rightarrow \quad k = \mathbf{\pm 4}$$

Q. 11. *What is the nature of roots of the quadratic equation $4x^2 - 12x + 9 = 0$?*

Sol. Comparing $4x^2 - 12x + 9 = 0$ with $ax^2 + bx + c = 0$ we get

$$a = 4$$

$$b = -12$$

$$c = 9$$

$$\therefore \quad b^2 - 4ac = (-12)^2 - 4(4)(9)$$

$$= 144 - 144 = 0$$

Since $b^2 - 4ac = 0$

$\therefore$ The roots are **real** and **equal.**

Q. 12. *Write the value of k for which the quadratic equation $x^2 - kx + 9 = 0$ has equal roots.*

(AI CBSE 2009 C)

Sol. Comparing $x^2 - kx + 9 = 0$ with $ax^2 + bx + c = 0$, we get

$$a = 1$$
$$b = -k$$
$$c = 9$$
$$\therefore \quad b^2 - 4ac = (-k)^2 - 4(1)(9)$$
$$= k^2 - 36$$

For equal roots,

$$b^2 - 4ac = 0$$
$$\Rightarrow \quad k^2 - 36 = 0 \quad \Rightarrow \quad k^2 = 36$$
$$\Rightarrow \quad k = \pm \mathbf{6}$$

Q. 13. *For what value of k are the roots of the quadratic equation $3x^2 + 2kx + 27 = 0$ real and equal?*

(CBSE 2008 C)

Sol. Comparing $3x^2 + 2kx + 27 = 0$ with $ax^2 + bx + c = 0$, we have:

$$a = 3$$
$$b = 2k$$
$$c = 27$$
$$\therefore \quad b^2 - 4ac = (2k)^2 - 4(3)(27)$$
$$= 4k^2 - (12 \times 27)$$

For the roots to be real and equal

$$b^2 - 4ac = 0$$
$$\Rightarrow \quad 4k^2 - (12 \times 17) = 0$$
$$\Rightarrow \quad 4k^2 = 12 \times 27$$
$$\Rightarrow \quad k^2 = \frac{12 \times 27}{4} = 81$$
$$\Rightarrow \quad k = \pm \mathbf{9}$$

Q. 14. *For what value of k are the roots of the quadratic equation $kx^2 + 4x + 1 = 0$ equal and real?*

(CBSE 2008 C)

Sol. Comparing $kx^2 + 4x + 1 = 0$, with $ax^2 + bx + c = 0$, we get

$$a = k$$
$$b = 4$$
$$c = 1$$
$$\therefore \quad b^2 - 4ac = (4)^2 - 4(k)(1)$$
$$= 16 - 4k$$

For equal and real roots, we have

$$b^2 - 4ac = 0$$
$$\Rightarrow \quad 16 - 4k = 0$$
$$\Rightarrow \quad 4k = 16$$
$$\Rightarrow \quad k = \frac{16}{4} = \mathbf{4}$$

Q. 15. *For what value of k does $(k - 12)\,x^2 + 2\,(k - 12)\,x + 2 = 0$ have equal roots?*

(AI CBSE 2008 C)

Sol. Comparing $(k - 12)\,x^2 + 2\,(k - 12)\,x + 2 = 0$ with $ax^2 + bx + c = 0$, we have:

$$a = (k - 12)$$
$$b = 2\,(k - 12)$$
$$c = 2$$

$\therefore \quad b^2 - 4ac = [2\,(k - 12)]^2 - 4\,(k - 12)\,(2)$
$= 4\,(k - 12)^2 - 8\,(k - 12)$
$= 4\,(k - 12)\,[k - 12 - 2]$
$= 4\,(k - 12)\,(k - 14)$

For equal roots,

$b^2 - 4ac = 0$

$\Rightarrow \quad 4\,(k - 12)\,[k - 14] = 0$

$\Rightarrow \quad$ Either $4\,(k - 12) = 0 \Rightarrow k = 12$

or $\quad k - 14 = 0 \Rightarrow k = 14$

But $k = 12$ makes $k - 12 = 0$ which is not required

$\therefore \quad k \neq 12$

$\Rightarrow \quad k = \mathbf{14}$

Q. 16. *For what value of k does the equation $9x^2 + 3kx + 4 = 0$ has equal roots?*

(AI CBSE 2008 C)

Sol. Comparing $9x^2 + 3kx + 4 = 0$ with $ax^2 + bx + c = 0$, we get

$$a = 9$$
$$b = 3k$$
$$c = 4$$

$\therefore \quad b^2 - 4ac = (3k)^2 - 4\,(9)\,(4)$
$= 9k^2 - 144$

For equal roots,

$b^2 - 4ac = 0$

$\Rightarrow \quad 9k^2 - 144 = 0$

$\Rightarrow \quad 9k^2 = 144$

$\Rightarrow \quad k^2 = \frac{144}{9} = 16$

$\Rightarrow \quad k = \pm\,\mathbf{4}$

II. SHORT ANSWER TYPE QUESTIONS

Q. 1. *Solve $2x^2 - 5x + 3 = 0$.*

Sol. We have:

$2x^2 - 5x + 3 = 0$ ...(1)

Comparing (1) with $ax^2 + bx + c = 0$,

$\therefore \quad a = 2$
$b = -5$
$c = 3$

$\therefore \quad b^2 - 4ac = (-5)^2 - 4\,(2)\,(3)$
$= 25 - 24 = 1$

Since, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore$ $x = \dfrac{-(-5) \pm \sqrt{1}}{2(2)}$

$= \dfrac{5 \pm 1}{4}$

Taking, + *ve* sign,

$$x = \frac{5+1}{4} = \frac{6}{4} = \frac{3}{2}$$

Taking, –*ve* sign,

$$x = \frac{5-1}{4} = \frac{4}{4} = 1$$

Thus, the required roots are

$$\mathbf{x = \frac{3}{2} \quad \text{and} \quad x = 1}$$

Q. 2. *Solve the following quadratic equation:*

$$2x^2 + 4x - 8 = 0$$

Sol. We have:

$$2x^2 + 4x - 8 = 0$$

Dividing by 2, we get

$$x^2 + 2x - 4 = 0 \quad \text{...(1)}$$

Comparing (1) with $ax^2 + bx + c = 0$,

$a = 1$,

$b = 2$

$c = -4$

$\therefore$ $b^2 - 4ac = (2)^2 - 4(1)(-4)$

$= 4 + 16 = 20$

Since, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore$ $x = \dfrac{-2 \pm \sqrt{20}}{2(1)}$

$\Rightarrow$ $x = \dfrac{-2 \pm 2\sqrt{5}}{2}$

$\Rightarrow$ $x = \dfrac{2\left[-1 \pm \sqrt{5}\right]}{2} = -1 \pm \sqrt{5}$

Taking +*ve* sign, we get

$$x = \left(-1 + \sqrt{5}\right)$$

Taking $-ve$ sign we get,

$$x = \left(-1-\sqrt{5}\right)$$

Thus, the required roots are $\boldsymbol{x = \left(-1+\sqrt{5}\right)}$ and $\boldsymbol{x = \left(-1-\sqrt{5}\right)}$.

Q. 3. *Solve:* $\dfrac{x+1}{x-1}+\dfrac{x-2}{x+2} = 3$. [CBSE 2012]

Sol. We have,

$$\frac{x+1}{x-1}+\frac{x-2}{x+2} = 3$$

$$\therefore \quad \frac{(x+2)(x+1)+(x-2)(x-1)}{(x-1)(x+2)} = 3$$

$$\Rightarrow \quad \frac{\left(x^2+2x+x+2\right)+\left(x^2-x-2x+2\right)}{x^2-x+2x-2} = 3$$

$$\Rightarrow \quad \frac{\left(x^2+3x+2\right)+\left(x^2-3x+2\right)}{x^2+x-2} = 3$$

$$\Rightarrow \quad x^2 + 3x + 2 + x^2 - 3x + 2 = 3(x^2 + x - 2)$$

$$\Rightarrow \quad 2x^2 + 4 = 3x^2 + 3x - 6$$

$$\Rightarrow \quad 3x^2 + 3x - 6 - 2x^2 - 4 = 0$$

$$\Rightarrow \quad x^2 + 3x - 10 = 0$$

$$\Rightarrow \quad x^2 + 5x - 2x - 10 = 0$$

$$\Rightarrow \quad x(x + 5) - 2(x + 5) = 0$$

$$\Rightarrow \quad (x + 5)(x - 2) = 0$$

Either $x + 5 = 0 \Rightarrow x = -5$

or $x - 2 = 0 \Rightarrow x = 2$

Thus, the required roots are

$$\boldsymbol{x = -5} \text{ and } \boldsymbol{x = 2}$$

Q. 4. *Solve (using quadratic formula):*

$$x^2 + 5x + 5 = 0$$

Sol. We have:

$$x^2 + 5x + 5 = 0$$

Comparing (1) with $ax^2 + bx + c = 0$, we have:

$$a = 1$$
$$b = 5$$
$$c = 5$$

$$\therefore \quad b^2 - 4ac = (5)^2 - 4(1)(5)$$
$$= 25 - 20 = 5$$

Since, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore$ $x = \dfrac{-5 \pm \sqrt{5}}{2(1)}$

$\Rightarrow$ $x = \dfrac{-5 \pm \sqrt{5}}{2}$

Taking +*ve* sign, we have:

$$x = \frac{-5 + \sqrt{5}}{2}$$

Taking –*ve* sign, we have:

$$x = \frac{-5 - \sqrt{5}}{2}$$

Thus, the required roots are:

$$x = \frac{-5 + \sqrt{5}}{2} \quad \textbf{and} \quad x = \frac{-5 - \sqrt{5}}{2}$$

Q. 5. *Solve for x:* $36x^2 - 12ax + (a^2 - b^2) = 0.$

Sol. We have:

$36x^2 - 12ax + (a^2 - b^2) = 0$ ...(1)

Comparing (1) with $Ax^2 + Bx + C = 0$, we have:

$$A = 36$$
$$B = -12a$$
$$C = (a^2 - b^2)$$

$$\therefore\ B^2 - 4AC = [-12a]^2 - 4(36)[a^2 - b^2]$$
$$= 144a^2 - 144(a^2 - b^2)$$
$$= 144a^2 - 144a^2 + 144b^2$$
$$= 144b^2$$

Since, $x = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$\therefore$ $x = \dfrac{-(-12a) \pm \sqrt{144b^2}}{2(36)}$

$\Rightarrow$ $x = \dfrac{12a \pm 12b}{72}$

Taking +*ve* sign, we have:

$$x = \frac{12a + 12b}{72}$$

$\Rightarrow$ $x = \dfrac{12}{72}[a + b] = \dfrac{1}{6}(a + b)$

Taking –*ve* sign, we get

$$x = \frac{12a - 12b}{72}$$

$$\Rightarrow \qquad x = \frac{12}{72}(a-b) = \frac{1}{6}(a-b)$$

Thus, the required roots are:

$$x = \frac{1}{6}(a+b) \quad \text{and} \quad x = \frac{1}{6}(a-b)$$

Q. 6. *Find the roots of the quadratic equation* $2x^2 - \sqrt{5}x - 2 = 0$, *using the quadratic formula.*

[NCERT Exemplar]

Sol. Comparing the given equation with the general equation $ax^2 + bx + c = 0$, we have

$$\left.\begin{aligned} a &= 2 \\ b &= -\sqrt{5} \\ c &= -2 \end{aligned}\right\} \Rightarrow \quad b^2 - 4ac = (-\sqrt{5})^2 - 4 \times 2 \times (-2)$$

$$= 5 + 16$$

$$= 21$$

Now, using the quadratic formula, we have:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \qquad x = \frac{-(-\sqrt{5}) \pm \sqrt{21}}{2(-2)}$$

$$= \frac{\sqrt{5} \pm 21}{4}$$

Taking the, positive sign, we get

$$x = \frac{\sqrt{5} + 21}{4}$$

Taking the negative sign, we get

$$x = \frac{\sqrt{5} - 21}{4}$$

Thus, $\qquad x = \dfrac{\sqrt{5} + 21}{4}$ and $\dfrac{\sqrt{5} - 21}{4}$

Q. 7. *Solve:* $16x^2 - 8a^2 x + (a^4 - b^4) = 0$ *for* x.

Sol. We have:

$$16x^2 - 8a^2 x + a^4 - b^4 = 0 \qquad \text{...(1)}$$

Comparing (1) with $ax^2 + bx + c = 0$, we get

$$a = 16$$

$$b = -8a^2$$

$$c = (a^4 - b^4)$$

$$\begin{aligned} \therefore \ b^2 - 4ac &= [-8a^2]^2 - 4\,(16)\,(a^4 - b^4) \\ &= 64\,a^4 - 64\,(a^4 - b^4) \\ &= 64\,a^4 - 64\,a^4 + 64\,b^4 \\ &= 64\,b^4 \end{aligned}$$

Since, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore$ $x = \dfrac{-(-8a^2) \pm \sqrt{64\,b^4}}{2\,(16)}$

$\Rightarrow$ $x = \dfrac{8a^2 \pm 8b^2}{32}$

$\Rightarrow$ $x = \dfrac{8\left[a^2 \pm b^2\right]}{32} = \dfrac{a^2 \pm b^2}{4}$

Now, taking +*ve* sign, we get

$$x = \frac{a^2 + b^2}{4}$$

Taking –*ve* sign, we get

$$x = \frac{a^2 - b^2}{4}$$

Thus, the required roots are:

$$\boldsymbol{x = \frac{a^2 + b^2}{4} \quad \text{and} \quad x = \frac{a^2 - b^2}{4}}$$

Q. 8. *Solve for x:* $9x^2 - 6ax + a^2 - b^2 = 0.$

Sol. We have:

$9x^2 - 6ax + (a^2 - b^2) = 0$ ...(1)

Comparing (1) with $ax^2 + bx + c = 0$, we get

$a = 9, \quad b = -6a \quad \text{and} \quad c = (a^2 - b^2)$

$\therefore\ b^2 - 4ac = (-6a)^2 - 4\,(9)\,(a^2 - b^2)$

$= 36a^2 - 36\,(a^2 - b^2)$

$= 36a^2 - 36a^2 + 36b^2$

$= 36b^2 = (6b)^2$

Since, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore$ $x = \dfrac{-(-6a) \pm \sqrt{(6b)^2}}{2\,(9)}$

$\Rightarrow$ $x = \dfrac{6a \pm 6b}{18}$

$\Rightarrow$ $x = \dfrac{6\left[a \pm b\right]}{18} = \dfrac{a \pm b}{3}$

Taking the +*ve* sign, we get

$$x = \frac{a+b}{3}$$

Taking the $-ve$ sign, we get

$$x = \frac{a-b}{3}$$

$\therefore$ The required roots are:

$$x = \boldsymbol{\frac{a+b}{3}} \quad \text{and} \quad x = \frac{\boldsymbol{a-b}}{3}$$

Q. 9. *Evaluate* $\sqrt{20+\sqrt{20+\sqrt{20+...}}}$

Sol. Let $\sqrt{20+\sqrt{20+\sqrt{20}}}... = x$

The given expression can be written as

$$x = \sqrt{20+\left(\sqrt{20+\sqrt{20}...}\right)}$$

or $$x = \sqrt{20+x}$$

squaring both side, we have

$$x^2 = \left(\sqrt{20+x}\right)^2$$

$$\Rightarrow \quad x^2 = 20 + x$$

$$\Rightarrow \quad x^2 - x - 20 = 0, \quad \text{where } x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Here : $a = 1$, $b = -1$ and $c = -20$

$$\therefore \quad x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 1 \times (-20)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{1+80}}{2} = \frac{1 \pm \sqrt{81}}{2} = \frac{1 \pm 9}{2}$$

Since the given expression is positive,

$\therefore$ Rejecting the negative sign, we have:

$$x = \frac{1+9}{2} = \frac{10}{2} = 5$$

Thus, $$\sqrt{20+\sqrt{20+\sqrt{20+...}}} = 5$$

Q. 10. *Using quadratic formula, solve the following quadratic equation for x:*

$$x^2 - 4ax + 4a^2 - b^2 = 0$$

Sol. Comparing the given equation with $ax^2 + bx + c = 0$, we have:

$$a = 1$$
$$b = -4a$$
$$c = 4a^2 - b^2$$

$$\therefore \; b^2 - 4ac = [-(4a)]^2 - 4(1)[4a^2 - b^2]$$
$$= 16a^2 - 4(4a^2 - b^2)$$
$$= 16a^2 - 16a^2 + 4b^2$$
$$= 4b^2 = (2b)^2$$

Since, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore \quad x = \frac{-(-4a) \pm \sqrt{(2b)^2}}{2(1)}$

$\Rightarrow \quad x = \frac{4a \pm 2b}{2}$

$\Rightarrow \quad x = \frac{2}{2}[2a \pm b] = 2a \pm b$

Taking the +*ve* sign, $x = 2a + b$

Taking the –*ve* sign, $x = 2a - b$

Thus, the required roots are:

$\boldsymbol{x = 2a + b}$ and $\boldsymbol{x = 2a - b}$

Q. 11. *Using quadratic formula, solve the following quadratic equation for x:*

$$x^2 - 2ax + (a^2 - b^2) = 0$$

Sol. Comparing the given equation with $ax^2 + bx + c = 0$, we have:

$a = 1, \quad b = -2a \quad$ and $\quad c = (a^2 - b^2)$

$\therefore \; b^2 - 4ac = (-2a)^2 - 4(1)(a^2 - b^2)$

$= 4a^2 - 4(a^2 - b^2)$

$= 4a^2 - 4a^2 + 4b^2 = 4b^2$

Since, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore \quad x = \frac{-(-2a) \pm \sqrt{4b^2}}{2(1)}$

$\Rightarrow \quad x = \frac{2a \pm 2b}{2}$

$\Rightarrow \quad x = \frac{2}{2}[a \pm b] = a \pm b$

Taking the +*ve* sign, we get

$x = a + b$

Taking the –*ve* sign, we get

$x = a - b$

Thus, the required roots are:

$\boldsymbol{x = a + b}$ and $\boldsymbol{x = a - b}$

Q. 12. *Find the roots of the equation:*

$$\frac{1}{x+3} + \frac{1}{2x-1} = \frac{11}{7x+9};\; x \neq -3, \frac{1}{2}, \frac{-9}{7}$$ (CBSE 2009 C)

Sol. We have:

$$\frac{1}{x+3} + \frac{1}{2x-1} = \frac{11}{7x+9}$$

$$\Rightarrow \quad \frac{2x - 1 + x + 3}{(x+3)(2x-1)} = \frac{11}{7x+9}$$

$$\Rightarrow \quad \frac{3x+2}{2x^2 - x + 6x - 3} = \frac{11}{7x+9}$$

$$\Rightarrow \quad \frac{(3x+2)(7x+9)}{2x^2+5x-3} = 11$$

$\Rightarrow$ $(3x + 2)(7x + 9) = 11(2x^2 + 5x - 3)$

$\Rightarrow$ $21x^2 + 27x + 14x + 18 = 22x^2 + 55x - 33$

$\Rightarrow$ $21x^2 + 41x + 18 = 22x^2 + 55x - 33$

$\Rightarrow$ $(21 - 22)x^2 + (41 - 55)x + (18 + 33) = 0$

$\Rightarrow$ $-x^2 + (-14x) + (51) = 0$

$\Rightarrow$ $x^2 + 14x - 51 = 0$

$\Rightarrow$ $x^2 + 17x - 3x - 51 = 0$

$\Rightarrow$ $x(x + 17) - 3(x + 17) = 0$

$\Rightarrow$ $(x + 17)(x - 3) = 0$

Either $x + 17 = 0 \Rightarrow x = -17$

or $x - 3 = 0 \Rightarrow x = 3$

Thus, the required roots of the given equation are:

3 and **– 17**

Q. 13. *Find the roots of the equation:*

$$\frac{1}{x} - \frac{1}{x-3} = \frac{4}{3};\ x \neq 0,\ 3$$ (CBSE 2009 C)

Sol. We have:

$$\frac{1}{x} - \frac{1}{x-3} = \frac{4}{3}$$

$$\Rightarrow \quad \frac{(x-3)-x}{x(x-3)} = \frac{4}{3}$$

$\Rightarrow$ $-3 \times 3 = 4 \times (x^2 - 3x)$

$\Rightarrow$ $-9 = 4x^2 - 12x$

$\Rightarrow$ $4x^2 - 12x + 9 = 0$ ...(1)

Comparing (1), with $ax^2 + bx + c = 0$, we get

$a = 4$

$b = -12$

$c = 9$

$\therefore$ $b^2 - 4ac = (-12)^2 - 4(4)(9)$

$= 144 - 144 = 0$

Now, the roots are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-(-12) \pm 0}{2(4)}$$

$$\Rightarrow \quad x = \frac{12}{8} = \frac{3}{2}$$

Thus, the roots are: $\mathbf{\frac{3}{2}}$ and $\mathbf{\frac{3}{2}}$

Q. 14. *Find the roots of the equation:*

$$\frac{1}{x-2} + \frac{1}{x} = \frac{8}{2x+5};\ x \neq 0, 2, \frac{-5}{2}$$ (CBSE 2008 C)

Sol. We have:

$$\frac{1}{x-2} + \frac{1}{x} = \frac{8}{2x+5}$$

$$\Rightarrow \frac{x+x-2}{x(x-2)} = \frac{8}{2x+5}$$

$$\Rightarrow \frac{2x-2}{x^2-2x} = \frac{8}{2x+5}$$

$$\Rightarrow (2x-2)(2x+5) = 8(x^2-2x)$$

$$\Rightarrow 4x^2 + 10x - 4x - 10 = 8x^2 - 16x$$

$$\Rightarrow -4x^2 + 22x - 10 = 0$$

$$\Rightarrow 2x^2 - 11x + 5 = 0 \quad ...(1)$$

Comparing (1) with $ax^2 + bx + c = 0$, we have:

$$a = 2$$

$$b = -11$$

$$c = 5$$

$$\therefore\ b^2 - 4ac = (-11)^2 - 4(2)(5)$$

$$= 121 - 40 = 81$$

Now, the roots are given by

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow x = \frac{-(-11) \pm \sqrt{81}}{2(2)}$$

$$= \frac{11 \pm 9}{4}$$

Taking the $+ve$ sign,

$$x = \frac{11+9}{4} = \frac{20}{4} = 5$$

Taking the $-ve$ sign,

$$x = \frac{11-9}{4} = \frac{2}{4} = \frac{1}{2}$$

Thus, the required roots are: **5 and $\frac{1}{2}$.**

Q. 15. *Find the roots of the following equation:*

$$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30};\ x = -4,\ 7$$ (CBSE 2012)

Sol. We have:

$$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$$

$$\Rightarrow \quad \frac{(x-7)-(x+4)}{(x+4)(x-7)} = \frac{11}{30}$$

$$\Rightarrow \quad \frac{x-7-x-4}{x^2-3x-28} = \frac{11}{30}$$

$$\Rightarrow \quad -11 \times 30 = 11 \times (x^2 - 3x - 28)$$

$$\Rightarrow \quad -30 = x^2 - 3x - 28$$

$$\Rightarrow \quad x^2 - 3x - 28 + 30 = 0$$

$$\Rightarrow \quad x^2 - 3x + 2 = 0$$

$$\Rightarrow \quad x^2 - 2x - x + 2 = 0$$

$$\Rightarrow x(x-2) - 1(x-2) = 0$$

$$\Rightarrow \quad (x-1)(x-2) = 0$$

Either $x - 1 = 0 \Rightarrow x = 1$

or $x - 2 = 0 \Rightarrow x = 2$

Thus, the required roots are: **1 and 2.**

Q. 16. *If α and β are roots of the equation $x^2 - 1 = 0$, form an equation whose roots are* $\frac{2\alpha}{\beta}$ and $\frac{2\beta}{\alpha}$.

Sol. $\because$ α and β are roots of $x^2 - 1 = 0$ and $x^2 - 1 = 0$ can be written as $x^2 + 0x - 1 = 0$ where $a = 1$, $b = 0$ and $c = -1$.

$\therefore$ Sum of roots $= \frac{-b}{a} = \frac{0}{1} = 0$

$$\Rightarrow \quad \alpha + \beta = 0$$

Also, product of roots $= \frac{c}{a} = \frac{-1}{1} = -1$

$$\Rightarrow \quad \alpha\beta = -1$$

Now, the roots of the new equation are $\frac{2\alpha}{\beta}$ and $\frac{2\beta}{\alpha}$

$\therefore$ Sum of the roots of the new equation $= \frac{2\alpha}{\beta} + \frac{2\beta}{\alpha}$

$$= \frac{2\alpha^2 + 2\beta^2}{\alpha\beta} = \frac{2(\alpha^2 + \beta^2)}{\alpha\beta}$$

$$= \frac{2[(\alpha+\beta)^2 - 2\alpha\beta]}{\alpha\beta}$$

$$= 2\left[\frac{(0)^2 - 2(-1)}{(-1)}\right] \quad \left| \because \alpha + \beta = 0 \text{ and } \alpha\beta = (-1) \right.$$

$$= 2\left[\frac{0+2}{(-1)}\right]$$

$$= 2 \times (-2) = -4$$

Product of the roots of the new equation $= \frac{2\alpha}{\beta} \times \frac{2\beta}{\alpha} = 4\frac{\alpha\beta}{\alpha\beta} = 4$

Since, a quadratic equation is given by

$$x^2 - \begin{pmatrix}\text{Sum of the} \\ \text{roots}\end{pmatrix} x + \begin{pmatrix}\text{Product} \\ \text{of the roots}\end{pmatrix} = 0$$

∴ The required quadratic equation is

$$x^2 - (-4)\, x + 4 = 0$$

or $$x^2 + 4x + 4 = 0$$

Remember

$$\alpha^2 + \beta^2 + 2\alpha\beta = (\alpha + \beta)^2$$

$$\Rightarrow \quad \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

III. LONG ANSWER TYPE QUESTIONS

Q. 1. *A two-digit number is 5 times the sum of its digits and is also equal to 5 more than twice the product of its digits. Find the number.*

Sol. Let the tens digit $= x$

And the ones digit $= y$

∴ The number $= 10x + y$

According to the conditions,

$$10x + y = 5\,(x + y) \quad \text{...(1)}$$

$$10x + y = 2xy + 5 \quad \text{...(2)}$$

From (1), we have

$$10x + y = 5x + 5y$$

$$\Rightarrow \quad 10x + y - 5x - 5y = 0$$

$$\Rightarrow \quad 5x - 4y = 0$$

$$\Rightarrow \quad 5x = 4y \quad \text{or} \quad y = \frac{5}{4}x$$

Substituting $y = \frac{5}{4}x$ in (2), we get

$$10x + \frac{5x}{4} = \frac{5x^2}{2} + 5$$

$$\Rightarrow \quad 40x + 5x = 10x^2 + 20 \quad \text{[Multiplying both sides by 4]}$$

$$\Rightarrow \quad 45x = 10x^2 + 20$$

$$\Rightarrow \quad 10x^2 - 45x - 20 = 0$$

$$\Rightarrow \quad 2x^2 - 9x - 4 = 0$$

$$\Rightarrow \quad 2x^2 - 8x - x + 4 = 0$$

$$\Rightarrow \quad 2x\,(x - 4) - 1\,(x - 4) = 0$$

$$\Rightarrow \quad (x - 4)\,(2x - 1) = 0$$

Either $x - 4 = 0 \Rightarrow x = 4$

or $2x - 1 = 0 \Rightarrow x = \frac{1}{2}$

But a digit cannot be a fraction,

∴ $x = 4 \Rightarrow$ The tens digit $= 4$

Now, the ones digit $y = \frac{5 \times 4}{4} = 5$

$\therefore \quad x = 4$ and $y = 5$

$\therefore$ The required number $= 10 \times 4 + 5$

$= 40 + 5 = \mathbf{45}$

Q. 2. *The denominator is one more than twice the numerator. If the sum of the fraction and its reciprocal is $2\frac{16}{21}$, find the fraction.*

Sol. Let the numerator be x

$\therefore \quad \text{Denominator} = (2x + 1)$

$\therefore \quad \text{Fraction} = \frac{x}{2x+1}$

Reciprocal of the fraction $= \frac{2x+1}{x}$

According to the condition,

$$\frac{x}{2x+1} + \frac{2x+1}{x} = 2\frac{16}{21}$$

$$\Rightarrow \frac{x \times x + (2x+1)(2x+1)}{x(2x+1)} = \frac{58}{21}$$

$$\Rightarrow \frac{x^2 + (2x+1)^2}{2x^2 + x} = \frac{58}{21}$$

$$\Rightarrow \frac{x^2 + 4x^2 + 4x + 1}{2x^2 + x} = \frac{58}{21}$$

$$\Rightarrow \frac{5x^2 + 4x + 1}{2x^2 + x} = \frac{58}{21}$$

$$\Rightarrow 21(5x^2 + 4x + 1) = 58(2x^2 + x)$$

$$\Rightarrow 105x^2 + 84x + 21 = 116x^2 + 58x$$

$$\Rightarrow 105x^2 - 116x^2 + 84x - 58x + 21 = 0$$

$$\Rightarrow -11x^2 + 26x + 21 = 0$$

$$\Rightarrow 11x^2 - 26x - 21 = 0 \quad \text{...(1)}$$

Comparing (1) with $ax^2 + bx + c = 0$, we have:

$a = 11$

$b = -26$

$c = -21$

$\therefore \quad b^2 - 4ac = (-26)^2 - 4 \times 11(-21)$

$= 676 + 924 = 1600$

Since, $\quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\therefore \quad x = \frac{-(-26) \pm \sqrt{1600}}{2(11)}$$

$$\Rightarrow \quad x = \frac{26 \pm 40}{22}$$

Taking the $+ve$ sign,

$$x = \frac{26+40}{22} = \frac{66}{22} = 3$$

Taking the $-ve$ sign,

$$x = \frac{26-40}{22} = \frac{-14}{22} = \frac{-7}{11}$$

But the numerator cannot be $\frac{-7}{11}$

$\therefore \quad x = 3 \Rightarrow$ The numerator = 3

$\therefore$ Denominator = 2 (3) + 1 = 7

Thus, the required fraction = $\mathbf{\frac{3}{7}}$

Q. 3. *The sum of a number and its reciprocal is $\frac{10}{3}$. Find the number.*

Sol. Let the required number = x

$\therefore$ Its reciprocal = $\frac{1}{x}$

According to the condition, we have:

$$[\text{The number}] + \begin{bmatrix}\text{Reciprocal of} \\ \text{the number}\end{bmatrix} = \frac{10}{3}$$

$$\Rightarrow \quad x + \frac{1}{x} = \frac{10}{3}$$

$$\Rightarrow \quad \frac{x^2+1}{x} = \frac{10}{3}$$

$$\Rightarrow \quad 3(x^2 + 1) = 10x$$

$$\Rightarrow \quad 3x^2 + 3 - 10x = 0$$

$$\Rightarrow \quad 3x^2 - 10x + 3 = 0$$

$$\Rightarrow \quad 3x^2 - 9x - x + 3 = 0$$

$$\Rightarrow \quad 3x(x-3) - 1(x-3) = 0$$

$$\Rightarrow \quad (x-3)(3x-1) = 0$$

Either $x - 3 = 0 \Rightarrow x = 3$

or $3x - 1 = 0 \Rightarrow x = \frac{1}{3}$

Thus, the required number is **3 or $\frac{1}{3}$**.

Q. 4. *The hypotenuse of a right triangle is $3\sqrt{10}$ cm. If the smaller side is tripled and the longer side doubled, new hypotenuse will be $9\sqrt{5}$ cm. How long are the sides of the triangle?*

Sol. Let the smaller side $= x$

$$\therefore \quad \text{Longer side} = \sqrt{(\text{Hypotenuse})^2 - (\text{smaller side})^2}$$

$$= \sqrt{\left(3\sqrt{10}\right)^2 - x^2}$$

$$= \sqrt{9 \times 10 - x^2}$$

$$= \sqrt{90 - x^2}$$

According to the condition, we have

$$[3\ (\text{Smaller side})]^2 + [2\ (\text{Longer side})]^2 = [\text{New Hypotenuse}]^2$$

$$\Rightarrow \quad [3\ (x)]^2 + \left[2\sqrt{(90 - x^2)}\right]^2 = \left[9\sqrt{5}\right]^2$$

$$\Rightarrow \quad 9x^2 + 4\ (90 - x^2) = 81 \times 5$$

$$\Rightarrow \quad 9x^2 + 360 - 4x^2 = 405$$

$$\Rightarrow \quad 5x^2 = 405 - 360$$

$$\Rightarrow \quad 5x^2 = 45$$

$$\Rightarrow \quad x^2 = \frac{45}{5} = 9$$

$$\Rightarrow \quad x = \pm\sqrt{9} = \pm 3$$

But $x = -3$ is not required, because the side of a triangle cannot be negative.

$$\therefore \quad x = 3 \quad \Rightarrow \quad \text{Smaller side} = 3 \text{ cm}$$

$$\therefore \quad \text{Longer side} = \sqrt{90 - 3^2}$$

$$= \sqrt{90 - 9}$$

$$= \sqrt{81} = 9 \text{ cm}$$

Thus, the required sides of the triangle are **3 cm** and **9 cm.**

Q. 5. *A motor-boat goes 10 km upstream and returns back to the starting point in 55 minutes. If the speed of the motor boat in still water is 22 km/hr, find the speed of the current.*

Sol. Let the speed of the current $= x$ km/hr

$\therefore$ The speed downstream $= (22 + x)$ km/hr

The speed upstream $= (22 - x)$ km/hr

$$\text{Since,} \quad \text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$\therefore$ Time for going 10 km downstream $= \left(\dfrac{10}{22 + x}\right)$ hrs

Time for returning back 10 km upstream $= \left(\dfrac{10}{22 - x}\right)$ hours

According to the condition,

$$\frac{10}{22+x} + \frac{10}{22-x} = \frac{55}{60}$$

$$\Rightarrow \quad 10\left[\frac{1}{22+x} + \frac{1}{22-x}\right] = \frac{11}{12} \qquad \left[\because \text{ 55 minutes} = \frac{55}{60} \text{ hours}\right]$$

$$\Rightarrow \quad 10 \times 12\left[\frac{22+x+22-x}{(22+x)(22-x)}\right] = 11$$

$$\Rightarrow \quad 120\left[\frac{44}{484-x^2}\right] = 11$$

$\Rightarrow \quad 11\ (484 - x^2) = 120 \times 44$

$\Rightarrow \quad 5324 - 11x^2 = 5280$

$\Rightarrow \quad 11x^2 = 5324 - 5280 = 44$

$$\Rightarrow \quad x^2 = \frac{44}{11} = 4$$

$\Rightarrow \quad x = \pm 2$

But speed of the current cannot be negative,

$\therefore \quad x = 2$

$\Rightarrow$ Speed of current = **2 km/hr**

Q. 6. *A motor boat whose speed in still water is 5 km/hr, takes 1 hour more to go 12 km upstream than to return downstream to the same spot. Find the speed of the stream.* (AI CBSE 2009 C)

Sol. Let the speed of the stream be x km/hr

$\therefore$ Downstream speed of the motor boat = $(x + 5)$ km/hr

$\Rightarrow$ Time taken to go 12 km upstream = $\frac{12}{5-x}$ hours

Time taken to return 12 km downstream = $\frac{12}{5+x}$ hours

According to the condition

$$\frac{12}{5-x} - \frac{12}{5+x} = 1$$

$\therefore \quad 12\ (5 + x) - 12\ (5 - x) = 1\ (5 - x)\ (5 + x)$

$\Rightarrow \quad 60 + 12x - 60 + 12x = 25 - x^2$

$\Rightarrow \quad 24x = 25 - x^2$

$\Rightarrow \quad x^2 + 24x - 25 = 0$

$\Rightarrow \quad x^2 + 25x - x - 25 = 0$

$\Rightarrow \quad x\ (x + 25) - 1\ (x + 25) = 0$

$\Rightarrow \quad (x - 1)\ (x - 25) = 0$

Either $x - 1 = 0 \Rightarrow x = 1$

or $x + 25 = 0 \Rightarrow x = -25$

But $x = -25$ is not admissible, because the speed of the stream cannot be negative.

$\therefore \quad x = 1$

$\Rightarrow$ speed of the stream = **1 km/hr.**

Q. 7. *Sum of the areas of two squares is 260 m^2. If the difference of their perimeters is 24 m, then find the sides of the two squares.* (AI CBSE 2009 C)

Sol. Let the side of one of the squares be 'x' metres

$\therefore$ Perimeter of square-I = $4 \times x$ metres = $4x$ metres

$\therefore$ Perimeter of square-II = $(24 + 4x)$ metres

$\therefore$ Side of the square-II = $\frac{1}{4}$ $(24 + 4x)$ metres = $(6 + x)$ metres

Now, according to the condition, we have:

$$x^2 + (6 + x)^2 = 260$$

$$\Rightarrow \quad x^2 + 36 + x^2 + 12x - 260 = 0$$

$$\Rightarrow \quad 2x^2 + 12x - 224 = 0$$

$$\Rightarrow \quad x^2 + 6x - 112 = 0 \quad \text{...(1)}$$

Comparing (1) with $ax^2 + bx + c = 0$, we get,

$$a = 1$$

$$b = 6$$

$$c = -112$$

$$\therefore \quad b^2 - 4ac = (6)^2 - 4\,(1)\,(-112)$$

$$= 36 + 448 = 484$$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-6 \pm \sqrt{484}}{2\,(1)}$$

$$= \frac{-6 \pm 22}{2}$$

Taking $+ve$ sign, we have

$$x = \frac{-6 + 22}{2} = \frac{16}{2} = 8$$

Taking $-ve$ sign, we have

$$x = \frac{-6 - 22}{2} = \frac{-28}{2} = -14$$

But $x = -14$ is not required, as the length of a side cannot be negative.

$\therefore \quad x = 8$

$\Rightarrow$ Side of square-I = **8 m**

$\Rightarrow$ Side of square-II = 6 + 8 m = **14 m.**

Q. 8. *The age of a father is twice the square of the age his son. Eight years hence, the age of the father will be 4 years more than three times the age of his son. Find their present ages.*

(AI CBSE 2009 C)

Sol. Let the present of son be 'x' years.

$\therefore$ Father's present age = $2x^2$ years

8 years hence:

Age of son = $(x + 8)$ years

Age of father = $(2x^2 + 8)$ years

According to the condition:

$$(2x^2 + 8) = 3(x + 8) + 4$$

$$\Rightarrow \quad 2x^2 + 8 - 3x - 24 - 4 = 0$$

$$\Rightarrow \quad 2x^2 - 3x + 8 - 28 = 0$$

$$\Rightarrow \quad 2x^2 - 3x - 20 = 0 \quad ...(1)$$

Comparing (1) with $ax^2 + bx + c = 0$, we get

$$a = 2$$

$$b = -3$$

$$c = -20$$

$$\therefore \quad b^2 - 4ac = (-3)^2 - 4(2)(-20)$$

$$= 9 + 160 = 169$$

Now, $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore \quad x = \frac{-(-3) \pm \sqrt{169}}{2(2)} = \frac{3 \pm 13}{4}$$

Taking $+ve$ sign,

$$x = \frac{3+13}{4} = \frac{16}{4} = 4$$

Taking $-ve$ sign,

$$x = \frac{3-13}{4} = \frac{-10}{4} = \frac{-5}{2}$$

But $x = \frac{-5}{2}$ is not required, as the age cannot be negative.

$$\therefore \quad x = 4$$

$\Rightarrow$ Present age of son = **4 years**

Present age of father = $\mathbf{2 \times 4^2 = 32}$ **years.**

Q. 9. *The diagonal of a rectangular field is 60 m more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.* (CBSE 2009 C)

Sol. *See Q-6 of Textbook Exercise 4.3.*

Q. 10. *A motor boat whose speed in still water is 16 km/h, takes 2 hours more to go 60 km upstream than to return to the same spot. Find the speed of the stream.* (CBSE 2009 C)

Sol. Let the speed of the stream = x km/hr

For the motor boat, we have:

$\therefore$ Downstream speed = $(16 + x)$ km/hr

Upstream speed = $(16 - x)$ km/hr

For going 60 km:

$$\text{Downstream} = \frac{60}{16 + x} \text{ hours}$$

$$\text{Upstream} = \frac{60}{16 - x} \text{ hours}$$

According to the condition:

$$\frac{60}{16 - x} - \frac{60}{16 + x} = 2$$

$\Rightarrow \quad 60(16 + x) - 60(16 - x) = 2(16 - x)(16 + x)$

$\Rightarrow \quad 960 + 60x - 960 + 60x = 2(256 - x^2)$

$\Rightarrow \quad 2x^2 + 120x = 2 \times 256 - 2x^2$

$\Rightarrow \quad x^2 + 60x = 256$

$\Rightarrow \quad x^2 + 60x - 250 = 0 \qquad ...(1)$

Comparing (1) with $ax^2 + bx + c = 0$,

$a = 1$

$b = 60$

$c = -256$

$\therefore \quad b^2 - 4ac = (60)^2 - 4(1)(-256)$

$= 3600 + 1024 = 4624$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-60 \pm \sqrt{4624}}{2(1)}$$

$$\Rightarrow \quad x = \frac{-60 \pm 68}{2}$$

Taking $+ve$ sign,

$$x = \frac{-60 + 68}{2} = \frac{8}{2} = 4$$

Taking $-ve$ sign,

$$x = \frac{-60 - 68}{2} = \frac{-128}{2} = -64$$

Since, the speed of a stream cannot be negative,

$\therefore \quad x = -64$ is not admissible

$\therefore \quad x = 4$

$\Rightarrow$ speed of the stream = **4 km/hr.**

Q. 11. *A train travels 288 km at a uniform speed. If the speed had been 4 km/hr more, it would have taken 1 hour less for the same journey. Find the speed of the train.* (CBSE 2009 C)

Sol. Let the speed of the train be x km/hr

$\because$ Total distance travelled = 288 km

$$\therefore \quad \text{Time taken} = \frac{288}{x} \text{ hours}$$

In the other case,

Speed of the train = $(x + 4)$ km/hr

$$\therefore \quad \text{Time taken} = \frac{288}{x + 4} \text{ hours}$$

According to the condition,

$$\frac{288}{x} - \frac{288}{x + 4} = 1$$

$$\Rightarrow \quad \frac{288(x + 4) - 288x}{x(x + 4)} = 1$$

$\Rightarrow$ $288x + 1152 - 288x = 1(x)(x + 4)$

$\Rightarrow$ $288x + 1152 - 288x = x^2 + 4x$

$\Rightarrow$ $1152 = x^2 + 4x$

$\Rightarrow$ $x^2 + 4x - 1152 = 0$ ...(1)

Comparing (1) with $ax^2 + bx + c = 0$,

$a = 1$

$b = 4$

$c = -1152$

$\therefore$ $b^2 - 4ac = (4)^2 - 4(1)(-1152)$

$= 16 + 4608$

$$\Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-(4) \pm \sqrt{4608}}{2(1)} = \frac{-4 \pm 68}{2}$$

Taking $+ve$ sign,

$$x = \frac{-4 + 68}{2} = \frac{64}{2} = 32$$

Taking $-ve$ sign,

$$x = \frac{-4 - 68}{2} = \frac{-72}{2} = -36$$

$\because$ speed cannot be negative,

$\therefore$ $x \neq -36$

$\therefore$ $x = 32$

$\Rightarrow$ speed of the train = **32 km/hr.**

Q. 12. *A train travels at a certain average speed for a distance of 63 km and then travels a distance of 72 km at an average speed of 6 km/hr more than its original speed of. If takes 3 hours to complete the total journey, what is its original average speed?* [NCERT Exemplar]

Sol. Let the original average speed = x km/hr

$\therefore$ Time taken to cover 72 km = $\frac{72}{x+6}$ hours

Time taken to cover 63 km = $\frac{63}{x}$ hours

Since, total time = 3 hours

$$\therefore \quad \frac{72}{x+6} + \frac{63}{x} = 3$$

$$\Rightarrow \quad \frac{1}{9}\left[\frac{72}{x+6} + \frac{63}{x}\right] = \frac{1}{9} \times 3$$

$\because$ *HCF of 72 and 63 is 9*

$$\Rightarrow \quad \frac{8}{x+6} + \frac{7}{x} = \frac{1}{3}$$

$$\Rightarrow \quad \frac{8x + 7(x+6)}{x(x+6)} = \frac{1}{3}$$

$$\Rightarrow \quad 8x + 7x + 42 = \frac{x^2+6x}{3}$$

$$\Rightarrow \quad 15x + 42 = \frac{x^2+6x}{3}$$

$\Rightarrow \quad 3[15x + 42] = x^2 + 6x$

$\Rightarrow \quad 45x + 126 - x^2 - 6x = 0$

$\Rightarrow \quad x^2 - 39x - 126 = 0$

$\Rightarrow \quad x^2 - 42x + 3x - 126 = 0$

$\Rightarrow \quad x(x - 42) + 3(x - 42) = 0$

$\Rightarrow \quad (x + 3)(x - 42) = 0$

Either $x + 3 = 0 \quad \Rightarrow \quad x = -3$

or $x - 42 = 0 \quad \Rightarrow \quad x = 42$

Since, speed cannot be negative,

$\therefore \quad x = -3$ is not desired.

Thus, the original speed of the train is 42 km/hr.

Q. 13. *If – 5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, then find the values of p and k.* (AI CBSE 2009)

Sol. Since – 5 is a root of $2x^2 + px - 15 = 0$,

$\therefore$ Substituting $x = -5$ in the given equation, we get

$2(-5)^2 + p(-5) - 15 = 0$

$\Rightarrow \quad 2(25) + (-5p) - 15 = 0$

$\Rightarrow \quad 50 - 5p - 15 = 0$

$\Rightarrow \quad -5p + 35 = 0$

$\Rightarrow \quad -5p = -35$

$$\Rightarrow \quad p = \frac{-35}{-5} = 7$$

Now, comparing the another quadratic equation $p(x^2 + x) + k = 0$, *i.e.*, $px^2 + px + k = 0$ with $ax^2 + bx + c = 0$, we have:

$a = p$

$b = p$

$c = k$

$b^2 - 4ac = p^2 - 4(p)(k)$

$\quad = p^2 - 4pk$

Since $p(x^2 - x) + k = 0$ has equal roots,

$\therefore \quad p^2 - 4pk = 0$

$\Rightarrow \quad (7)^2 - 4(7)k = 0 \qquad | \because p = 7$

$\Rightarrow \quad 49 - 28k = 0$

$$\Rightarrow \quad k = \frac{-49}{-28} = \frac{7}{4}$$

Thus, the required values of

$$\boldsymbol{p = 7 \text{ and } k = \frac{7}{4}}.$$

Q. 14. *In a class test, the sum of Gagan's marks in Mathematics and English is 45. If he had 1 more mark in Mathematics and 1 less in English, the product of marks would have been 500. Find the original marks obtained by Gagan in Mathematics and English separately.* (AI CBSE 2008 C)

Sol. Let Gagan's marks in maths $= x$

and Marks in English $= (45 - x)$

$\therefore$ According to the condition,

$$(x + 1) \times (45 - x + 1) = 500$$
$$\Rightarrow \quad (x + 1) \times (44 - x) = 500$$
$$\Rightarrow \quad 44x - x^2 + 44 - x = 500$$
$$\Rightarrow \quad -x^2 + 44x - 456 - x = 0$$
$$\Rightarrow \quad x^2 - 43x + 456 = 0$$
$$\Rightarrow \quad x^2 - 19x - 24x + 456 = 0$$
$$\Rightarrow \quad x(x - 19) - 24(x - 19) = 0$$
$$\Rightarrow \quad (x - 19)(x - 24) = 0$$

$\because$ $24 \times 19 = 45$
$-43 = (-24) + (-19)$

Either $x - 19 = 0 \Rightarrow x = 19$

or $x - 24 = 0 \Rightarrow x = 24$

When $x = 19$, then $45 - 19 = 26$

When $x = 24$, then $45 - 24 = 21$

$\therefore$ Gagan's marks in **Maths = 19 and in English = 26**

Or

Gagan's marks in **Maths = 24 and in English = 21.**

Q. 15. *The sum of areas of two squares is 640 m^2. If the difference of their perimeters is 64 m, find the sides of two squares.* (CBSE 2008 C)

Sol. Let the side of square I be x metres.

$\therefore$ Perimeter of square I $= 4x$ metres

$\Rightarrow$ Perimeter of square II $= (64 + 4x)$ m

$\therefore$ Side of square II $= \frac{1}{4}(64 + 4x)$ m $= (16 + x)$ m

Now Area of square I $= x \times x = x^2$

Area of square II $= (16 + x) \times (16 + x) = (16 + x)^2 = 256 + x^2 + 32x$

According to the condition,

$$\begin{bmatrix}\text{Area of}\\ \text{square I}\end{bmatrix} + \begin{bmatrix}\text{Area of the}\\ \text{square II}\end{bmatrix} = 640$$
$$\Rightarrow \quad x^2 + [256 + x^2 + 32x] = 640$$
$$\Rightarrow \quad x^2 + x^2 + 32x + 256 - 640 = 0$$
$$\Rightarrow \quad 2x^2 + 32x - 384 = 0$$
$$\Rightarrow \quad x^2 + 16x - 192 = 0$$
$$\Rightarrow \quad x^2 + 24x - 8x - 192 = 0$$
$$\Rightarrow \quad x(x + 24) - 8(x + 24) = 0$$

$\because$ $24 - 8 = 16$
and $24 \times 8 = 192$

$\Rightarrow \quad (x + 24)(x - 8) = 0$

Either $x + 24 = 0 \Rightarrow x = -24$

or $x - 8 = 0 \Rightarrow x = +8$

$\because$ side of a square cannot be negative,

$\therefore$ Rejecting $x = -24$, we have $x = 8$

$\Rightarrow$ Side of smaller square = **8 m**

Side of larger square = 8 + 16 m = **24 m.**

Q. 16. *In a class test, the sum of Kamal's marks in Mathematics and English is 40. Had he got 3 marks more in Mathematics and 4 marks less in English, the product of his marks would have been 360. Find his marks in two subjects separately.* (CBSE 2012)

Sol. Let Kamal's marks in Maths = x

$\therefore$ His marks in English = $(40 - x)$

According to the condition,

$$(x + 3)[40 - x - 4] = 360$$
$$\Rightarrow (x + 3)(36 - x) = 360$$
$$\Rightarrow 36x - x^2 + 108 - 3x - 360 = 0$$
$$\Rightarrow -x^2 + 33x - 252 = 0$$
$$\Rightarrow x^2 - 33x + 252 = 0$$
$$\Rightarrow x^2 - 21x - 12x + 252 = 0$$
$$\Rightarrow x(x - 21) - 12(x - 21) = 0$$
$$\Rightarrow (x - 21)(x - 12) = 0$$

$\because$ *$(-21) \times (-12) = 252$*
$(-21) + (-12) = -33$

Either $(x - 21) = 0 \Rightarrow x = 21$

or $(x - 12) = 0 \Rightarrow x = 12$

For $x = 21$, Marks of Kamal

in Maths = **21**

in English = 40 – 21 = **19**

For $x = 12$, Marks of Kamal

in Maths = **12**

in English = 40 – 12 = **28.**

Q. 17. *Find the value of p for which the quadratic equation $4x^2 + px + 3 = 0$ has equal roots.* (AI CBSE 2014)

Hint:

For equal roots, $b^2 - 4ac = 0$

$\therefore$ $p^2 - 4(4)(3) = 0$ *or* $p^2 - 48 = 0 \Rightarrow p = \pm\sqrt{48} = \pm 4\sqrt{3}$

Q. 18. *Solve the quadratic equation $2x^2 + ax - a^2 = 0$* [AI CBSE (Delhi) 2014]

Sol.
$$2x^2 + ax - a^2 = 0$$
$$\Rightarrow 2x^2 + 2ax - ax - a^2 = 0$$
$$\Rightarrow 2x[x + a] - a[x + a] = 0$$
$$\Rightarrow (x + a)(2x - a) = 0$$

splitting 'ax' into '$2ax$' and '$-ax$'

$$x = -a \text{ or } x = \frac{a}{2}$$

IV. HOTS QUESTIONS

Q. 1. *Had Ravita scored 10 more marks in her Mathematics test out of 30 marks, 9 times these marks would have been the square of her actual marks. How many marks did she get in the test?*

[NCERT Exemplar]

Sol. Let actual marks be x

$\therefore$ 9 × [Actual marks + 10] = [Square of actual marks]

or $9 \times (x + 10) = x^2$

$\Rightarrow 9x + 90 = x^2$

$\Rightarrow x^2 - 9x - 90 = 0$

$\Rightarrow x^2 - 15x + 6x - 90 = 0$

$\Rightarrow x(x - 15) + 6(x - 15) = 0$

$\Rightarrow (x + 6)(x - 15) = 0$

Either $x + 6 = 0 \Rightarrow x = -6$

or $x - 15 = 0 \Rightarrow x = 15$

But marks cannot be less than 0.

$\therefore x = -6$ is not desired.

Thus, Ravita got 15 marks in her Mathematics test.

Q. 2. *A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.* (AI CBSE 2008)

Sol. Let the speed of the stream = x km/hr

$\therefore$ **speed of the motor boat:**

upstream = $(18 - x)$ km/hr

downstream = $(18 + x)$ km/hr

$\Rightarrow$ Time taken by the motor boat in going:

24 km downstream = $\frac{24}{18 + x}$ hours

$\because Time = \frac{Distance}{Speed}$

24 km upstream = $\frac{20}{18 - x}$ hours

According to the condition:

$$\frac{24}{18 - x} - \frac{24}{18 + x} = 1$$

$\Rightarrow 24 \times (18 + x) - 2(18 - x) = 1(18 - x)(18 + x)$

$\Rightarrow 24[18 + x - 18 + x] = 18^2 - x^2$

$\Rightarrow 24[2x] = 324 - x^2$

$\Rightarrow 48x = 324 - x^2$

$\Rightarrow x^2 + 48x - 324 = 0$

$\Rightarrow x^2 + 54x - 6x - 324 = 0$

$\Rightarrow x(x + 54) - 6(x + 54) = 0$

$\Rightarrow (x - 6)(x + 54) = 0$

Either $x - 6 = 0 \Rightarrow x = 6$

or $x + 54 = 0 \Rightarrow x = -54$

But speed cannot be negative

$\therefore$ Rejecting $x = -54$, we have

$x = 6 \Rightarrow$ Speed of the boat = **6 km/hr.**

Q. 3. *In a class test, the sum of marks obtained by P in Mathematics and Science is 28. Had he got 3 more marks in Mathematics and 4 marks less in Science, the product of marks obtained in the two subjects would have been 180? Find the marks obtained in two subjects separately.* (CBSE 2008)

Sol. Let marks obtained by P in Maths be 'x'.

$\therefore$ His marks in Science = $(28 - x)$

According to the condition,

$$(x + 3)(28 - x - 4) = 180$$
$$\Rightarrow (x + 3)(-x + 24) = 180$$
$$\Rightarrow 24x - x^2 + 72 - 3x = 180$$
$$\Rightarrow -x^2 + 21x + 72 - 180 = 0$$
$$\Rightarrow -x^2 + 21x - 108 = 0$$
$$\Rightarrow x^2 - 21x + 108 = 0$$
$$\Rightarrow x^2 - 12x - 9x + 108 = 0$$
$$\Rightarrow x(x - 12x) - 9(x - 12) = 0$$
$$\Rightarrow (x - 9)(x - 12) = 0$$
$$\Rightarrow (x - 9)(x - 12) = 0$$

Either $x - 9 = 0 \Rightarrow x = 9$

or $x - 12 = 0 \Rightarrow x = 12$

When $x = 9$ then $28 - x = 28 - 9 = 19$

When $x = 12$ then $28 - x = 28 - 12 = 16$

Thus P's marks in **Maths = 9** and **Science = 19**

Or

P's marks in **Maths = 12** and **Science = 16.**

Q. 4. *Solve for x:*

$\sqrt{3}x^2 - 2\sqrt{2x} - 2\sqrt{3} = 0$ [CBSE (Foreign) 2014]

Hint:

$\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$, we have

$a = \sqrt{3},\ b = \left(-2\sqrt{2}\right)$ *and* $c = \left(-2\sqrt{3}\right)$

$\therefore\ b^2 - 4ac = \left(-2\sqrt{2}\right)^2 - 4\left(\sqrt{3}\right)\left(-2\sqrt{3}\right) = \sqrt{32}$

Using Quadratic formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

we get, $x = \sqrt{6}$

Q. 5. *Solve for x :*

$x^2 + 5\sqrt{5}x - 70 = 0$

Hint:

$a = 1,\ b = 5\sqrt{5}$ *and* $c = -70$

$\Rightarrow\ b^2 - 4ac = \left(5\sqrt{5}\right)^2 - 4(1)(-70) = 9\sqrt{5}$

Now use quad. formula to get $x = 2\sqrt{5}, -7\sqrt{x}$

Q. 6. *At 't' minutes past 2 pm, the time needed by the minute hand of a clock to show 3 pm was found to be 3 minutes less than $\frac{t^2}{4}$ minutes. Find 't'.*

Sol. For a minute-hand time needed to show 2 pm to 3 pm is '60' minutes.

It has already covered 't' minutes.

∴ Time required by the minute-hand to reach to 12 (at 3 pm) = (60 – t) minutes.

$$\therefore \quad \left(\frac{t^2}{4} - 3\right) = (60 - t)$$

$$\Rightarrow \quad \frac{t^2}{4} + t - 63 = 0$$

$$\Rightarrow \quad t^2 + 4t - 252 = 0$$

Solving, we get, $t = 14$ or -18

But $t = -18$ is not desirable (being negative)

Thus, t = **14 minutes.**

Q. 7. *A train, travelling at a uniform speed for 360 km, would have taken 48 minutes less to travel the same distance if its speed were 5 km/hr more. Find the original speed of the train.*

[NCERT Exemplar]

Sol. Let the original speed be x km/hr

∴Original time taken = $\frac{360}{x}$ hours

New speed = $(x + 5)$ km/hr

∴ New time = $\frac{360}{x+5}$ km / hr

According to the condition,

$$\frac{360}{x} = \frac{360}{x+5} + \frac{48}{60}$$

$$\Rightarrow \quad 360\left[\frac{x+5-x}{x(x+5)}\right] = \frac{4}{5}$$

$$\Rightarrow \quad 360\left[\frac{5}{x^2+5x}\right] = \frac{4}{5} \Rightarrow x^2 + 5x - 2250 = 0$$

Solving for x, we get x = – 50 or 45

Speed cannot be negative

∴ Rejecting x = – 50, we have x = 45

Thus, the original speed of the train = **45 km/hr.**

Q. 8. *If the roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal, then prove that $2b = a + c$*

Hint: *For equal roots D = 0*

Here, $D = (c - a)^2 - 4(b - c)(a - b)$

$\therefore \quad (c - a)^2 - 4(b - c)(a - b) = 0$

$\Rightarrow \quad c^2 + a^2 + 4b^2 - 2ac - 4ab + 4ac - 4bc = 0$

$\Rightarrow \quad (c + a - 2b)^2 = 0$

$\Rightarrow \quad (c + a - 2b) = 0 \quad \Rightarrow c + a = 2b$ *(Hence Proved)*

Q. 9. *If the roots of the equations*

$ax^2 + 2bx + c = 0$ *and* $bx^2 - 2\sqrt{ac}\,x + b = 0$

are simultanously real then prove that $b^2 = 4ac$

Hint: *Let* D_1 *and* D_2 *be the discriminants*

$\therefore$ $D_1 \geq 0$ *and* $D_2 \geq b^2$

$\Rightarrow$ $4b^2 - 4ac \geq 0$ *and* $4ac - 4b^2 \geq 0$

$\Rightarrow$ $b^2 \geq ac$ *and* $ac \geq b^2$

$b^2 = ac$

Q. 10. *If the roots of the equation*

$(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$ *are equal, then prove that*

either $a = 0$ *or* $a^3 + b^3 + c^3 = 3abc$

Hint:

$$D = b^2 - 4ac$$
$$= [-2(a^2 - bc)]^2 - 4\,(a^2 - ab)\,(b^2 - ac)$$
$$= 4a\,(a^3 + b^3 + c^3 - 3abc)$$

For equal roots, $D = 0$

$\therefore$ $4a\,(a^3 + b^3 + c^3 - 3abc) = 0$

$\Rightarrow$ $a = 0$ *or* $a^3 + b^3 + c^3 = 3abc$

TEST YOUR SKILLS

1. A natural number, when increased by 12, equals 160 times its reciprocal. Find the number. [NCERT Exemplar]

Hint: *Let the natural number be x*

$\therefore$ $x + 12 = \dfrac{160}{x}$

$\Rightarrow$ $x(x + 12) = 160$

$\Rightarrow$ $x^2 + 12x - 160 = 0$

2. By increasing the list price of a book by ₹ 10, a person can buy 10 books less for ₹ 1200. Find the original list price of the book. (CBSE 2007)
3. The hypotenuse of a right-angled triangle is 1 cm more than twice the shortest side. If the third side is 2 cm less than the hypotenuse, find the sides of the triangle. (CBSE 2007)
4. A passenger train takes 2 hours less for a journey of 300 km, if its speed is increased by 5 km/hr from its usual speed. Find its usual speed. (CBSE 2006, 2007)
5. The numerator of a fraction is one less than its denominator. If three is added to each of the numerator and denominator, the fraction is increased by $\frac{3}{28}$. Find the fraction. (AI CBSE 2007)
6. The difference of squares of two natural numbers is 45. The square of the smaller number is four times the larger number. Find the numbers. (AI CBSE 2007)
7. Solve: $x^2 + 5\sqrt{5}x - 70$ [NCERT Exemplar]

8. A train travels a distance of 300 km at a uniform speed. If the speed of the train is increased by 5 km an hour, the journey would have taken two hours less. Find the original speed of the train. (CBSE 2006)

9. The speed of a boat in still water is 11 km/hr. It can go 12 km upstream and returns downstream to the original point in 2 hours 45 minutes. Find the speed of the stream. (AI CBSE 2006)

10. Determine the value of k for which the quadratic equation $4x^2 - 3kx + 1 = 0$ has equal roots. (CBSE 2006 C)

11. Using quadratic formula, solve the following equation for 'x':

$$ab\,x^2 + (b^2 - ac)\,x - bc = 0$$ (AI CBSE 2006 C)

12. The sum of the numerator and the denominator of a fraction is 12. If the denominator is increased by 3, the fraction becomes $\frac{1}{2}$. Find the fraction. (AI CBSE 2006 C)

13. Rewrite the following as a quadratic equation in x and then solve for x:

$$\frac{4}{x} - 3 = \frac{5}{2x+3}; \quad x \neq 0, -\frac{3}{2}$$ (AI CBSE 2006 C)

14. A two digit number is such that the product of its digits is 18. When 63 is subtracted from the number, the digits interchange their places. Find the number. (AI CBSE 2006 C)

15. A train covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hr more, it would have taken 30 minutes less for the journey. Find the original speed of the train. (AI CBSE 2006 C)

16. Solve for x:

$$\frac{x+1}{x-1} + \frac{x-2}{x+2} = 3; \quad (x \neq 1, -2)$$ (CBSE 2012)

17. Using quadratic formula, solve the following for x:

$$9x^2 - 3\,(a^2 + b^2)\,x + a^2\,b^2 = 0$$ (CBSE 2012)

18. Find the equation whose roots are reciprocal of the roots of

$$3x^2 - 5x + 7 = 0$$

19. A number consists of two digits whose product is 18. When 27 is subtracted from the number, the digits change their places. Find the number. (AI CBSE 2005 C)

20. A number consisting of two digits is seven times the sum of its digits. When 27 is subtracted from the number, the digits are reversed. Find the number. (AI CBSE 2005 C)

21. The sum of the squares of two consecutive odd numbers is 394. Find the nimbers. [CBSE (Foreign) 2014]

Hint:

Let the two consecutive odd numbers be x and x + 2.

$\therefore \quad x^2 + (x+2)^2 = 394$

$\Rightarrow \quad x^2 + x^2 + 4x + 4 = 394$

$\Rightarrow \quad 2x^2 + 4x - 390 = 0$

$\Rightarrow \quad x^2 + 2x - 195 = 0$

$\Rightarrow \quad x^2 + 15x - 13x - 195 = 0 \quad$ *or* $\quad x(x+15) - 13(x+15) = 0$

$\Rightarrow \quad x = 13 \quad$ *or* $\quad x = -15$

$\therefore$ *For* $x = 13,\ x + 2 = 13 + 2 = 15$

Thus, the required numbers are **13** and **15**.

22. An aeroplane left 30 minutes later than its scheduled time and in order to reach its destination 1500 km away in time, it has to increase its speed by 250 km/hr from its usual speed. Determine its usual speed. (AI CBSE 2005 C)

23. Using quadratic formula, solve for x:

$$9x^2 - 3(a + b)x + ab = 0 \quad \text{(CBSE 2012)}$$

24. Find the number which exceeds its positive square root by 20.

25. The sum of two numbers is 15 and the sum of their reciprocals is $\frac{1}{3}$. Find the numbers. (CBSE 2005)

26. A two digit number is such that the product of its digits is 14. If 45 is added to the number, the digits interchange their places. Find the number. (AI CBSE 2005)

27. A two digit number is such that the product of its digits is 20. If 9 is added to the number, the digits interchange their places. Find the number. (AI CBSE 2005)

28. A two digit number is such that the product of its digits is 15. If 8 is added to the number, the digits interchange their places. Find the number. (AI CBSE 2005)

29. Solve for x:

$$\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}; \quad (x \neq 2, 4) \quad \text{(AI CBSE 2005)}$$

30. Solve for x:

$$ab\,x^2 + (b^2 - ac)\,x - bc = 0 \quad \text{(AI CBSE 2005)}$$

31. The sum of two numbers is 18. The sum of their reciprocals is $\frac{1}{4}$. Find the numbers. (AI CBSE 2005)

32. The sum of two numbers is 16, and the sum of their reciprocals is $\frac{1}{3}$. Find the numbers. (AI CBSE 2005)

33. The sum of two numbers 'a' and 'b' is 15, and sum of their reciprocals $\frac{1}{a}$ and $\frac{1}{b}$ is $\frac{3}{10}$. Find the numbers 'a' and 'b'. (CBSE 2005)

34. Solve for x:

$$\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}; \quad a \neq 0, \quad b \neq 0, \quad x \neq 0 \quad \text{(CBSE 2012)}$$

35. Find the roots of the following quadratic equation:

$$\frac{2}{5}x^2 - x - \frac{3}{5} = 0 \quad \text{(CBSE Sample Paper 2011)}$$

Hint:

$$\frac{2}{5}x^2 - x - \frac{3}{5} = 0 \quad \Rightarrow \quad 2x^2 - 5x - 3 = 0$$

$$\Rightarrow \quad (2x + 1)(x - 3) = 0$$

$$\Rightarrow \quad x = -\frac{1}{2} \text{ and } x = 3$$

36. Find the roots of the equation:

$$\frac{1}{2x-3}+\frac{1}{x-5} = 1; \quad x \neq \frac{3}{2}, 5$$ (CBSE Sample Paper 2011)

37. A natural number when subtracted from 28, becomes equal to 160 times its reciprocal. Find the number. (CBSE Sample Paper 2011)

38. Find two consecutive odd positive integers, sum of whose squares is 290.

(CBSE Sample Paper 2011)

39. Find the values of k for which the quadratic equation (CBSE Delhi 2014)

$(k + 4)\ x^2 + (k + 1)\ x + 1 = 0$ has equal roots. Also find these roots.

Hint:

From $(k + 4)x^2 + (k + 1)x + 1 = 0$, *we get*

$$a = (k + 4), \quad b = (k + 1) \text{ and } c = 1$$

$\therefore$ $b^2 - 4ac = (k + 1)^2 - 4\ (k + 4)\ (1)$

$= k^2 - 2k - 15$

For equal roots, $b^2 - 4ac = 0$

$\therefore$ $k^2 - 2k - 15 = 0$

$\Rightarrow$ $k = 5$ *or* $k = -3$

For $k = 5$, *we have* $(k + 4)x^2 + (k + 1)\ x + 1 = 0$

$\equiv$ $9x^2 + 6x + 1 = 0$

Solving for x, we get $x = \frac{-1}{3}, \frac{-1}{3}$

For $k = -3$, *we have*

$$x^2 - 2x + 1 = 0$$

$\therefore$ *solving it, we get* $x = 1, x = 1$

40. Solve for x :

$$\frac{16}{x}-1 = \frac{15}{x+1} \quad ; \quad x \neq 0, -1.$$ (AI CBSE 2014)

Hint:

$$\frac{16}{x}-1 = \frac{15}{x+1} \Rightarrow \frac{16}{x}-\frac{15}{x+1}=1$$

or $16(x + 1) - 15(x) = x(x + 1) \Rightarrow x^2 = 16$

$\therefore$ $x = \pm 4$

41. Solve for x :

$$\frac{x-2}{x-3}+\frac{x-4}{x-5}=\frac{10}{3}; \quad x \neq 3, 5$$ (AI CBSE 2014)

Hint:

$$L.H.S. = \frac{x-2}{x-3}+\frac{x-4}{x-5} \Rightarrow \frac{(x-5)(x-2)+(x-4)(x-3)}{(x-3)(x-5)} \quad \text{(By cross multiplication)}$$

$$\Rightarrow \quad \frac{2x^2-14x+22}{x^2-8x+15}$$

$$\therefore \quad \frac{2x^2-14x+22}{x^2-8x+15}=\frac{10}{3} \quad or \quad 3[2x^2 - 14x + 22] = 10[x^2 - 8x + 15]$$

On simplification, we get $2x^2 - 19x + 42 = 0 \Rightarrow x = \frac{7}{2}$

ANSWERS

Test Your Skills

1. 8	**2.** ₹ 30	**3.** 8 cm, 15 cm, 17 cm	**4.** 25 km/hr
5. $\frac{3}{4}$	**6.** 9 and 6	**7.** $7\sqrt{5}; 2\sqrt{5}$	**8.** 25 km/hr
9. 5 km/hr	**10.** $k \leq \frac{4}{3}$ or $k \ \frac{4}{3}$	**11.** $x \ \frac{c}{b}, x \ \frac{b}{a}$	**12.** $\frac{5}{7}$
13. –2, 1	**14.** 92	**15.** 45 km/hr	**16.** $x = -5, 2$
17. $\frac{a^2}{3}, \frac{b^2}{3}$	**18.** $7x^2 - 5x + 3 = 0$	**19.** 63	**20.** 63
21. 14	**22.** 750 km/h	**23.** $x \ \frac{a}{3}, \frac{b}{3}$	**24.** $x = 16$
25. 5, 10	**26.** 27	**27.** 45	**28.** 35
29. $\frac{5}{2}, 5$	**30.** $x = -b$	**31.** 6, 12	**32.** 12, 14
33. a = 5 or 10, b = 10 or 5		**34.** x = –a	**35.** $x = \frac{1}{2}, 3$
36. $4 \ \frac{3\sqrt{2}}{2}$	**37.** 8	**38.** 11 and 13	